Simple Harmonic Motion - SHM

Purpose: to determine the relationship between the period of oscillation of a mass on a spring to:

a) the mass
b) variable of your choice

Hypothesis:

a)

$T = 2\pi$ root (m/k)

T is the time for the mass to do one full oscillation

m is the mass

k is the spring constant, k=F/x eg. 0.5kg x 9.8N/kg/0.050m, the distance the spring extends

b) don't change k, find another variable and guess a hypothesis

Procedure/analysis

- determine k
- graph T vs root m
- graph T vs your variable

don't forget uncertainties

calculate % deviation from your slope and theory

Conclusion
sources of uncertainty

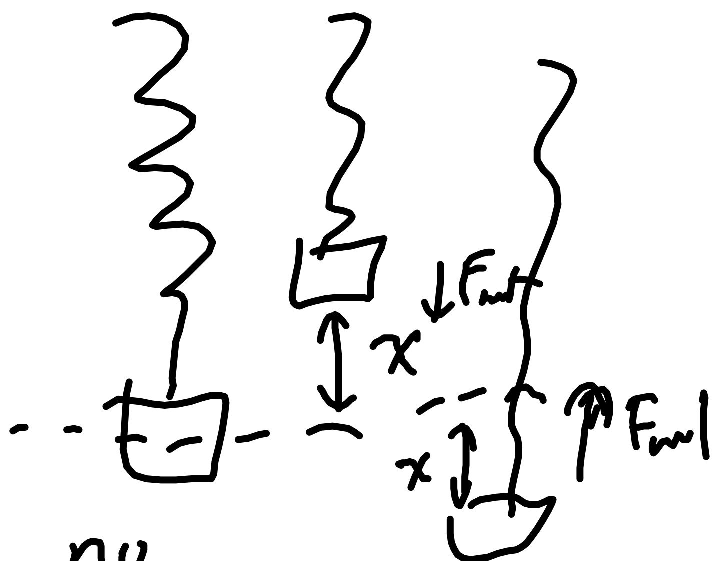
due next Tuesday

Simple Harmonic Motion, SHM

Define:

Motion with a restoring force towards "rest" position, with the force proportional to the deviation from rest position.

eg. Mass on a spring or pendulum

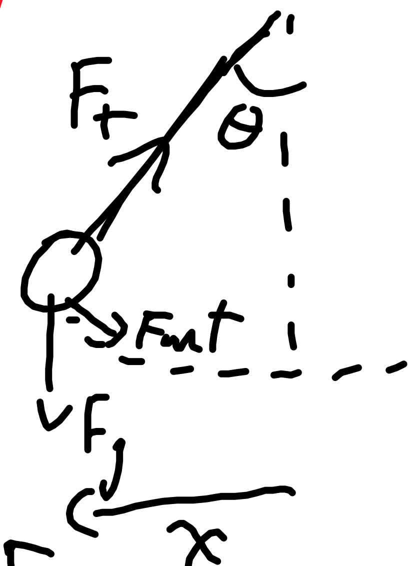


no a

$F_e = F_g$

$F_{net} = 0$

$F_{net} = -kx$



$F_{restoring} = -kx$

true for small θ

$k = \underline{\qquad}$

$ma = -kx$

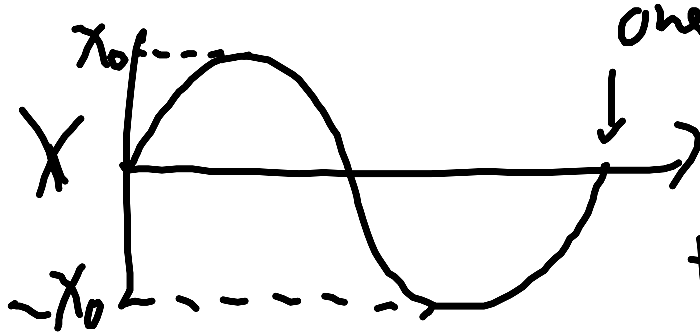
$a = -\frac{k}{m} x$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$x = A\sin(\omega t + \phi)$$

Amplitude = max $x = x_{max} = x_0$

one period T



$$\omega = 2\pi f = 2\pi \frac{1}{T}$$

ω is the angular frequency

$$\frac{dx}{dt} = \frac{d\, x_0 \sin(\omega t)}{dt}$$

$$\frac{dx}{dt} = \omega x_0 \cos(\omega t)$$

$$v = \omega x_0 \cos(\omega t)$$

$$V_{max} = \omega x_0$$

$$V = \omega\sqrt{x_0^2 - x^2}$$

velocity at any position, x

$$\frac{dv}{dt} = \frac{d\,\omega x_0 \cos(\omega t)}{dt}$$

$$a = -\omega^2 x_0 \sin(\omega t)$$

$$a = -\frac{k}{m}x \qquad a_{max} = -\frac{k}{m}x_0$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \frac{\sqrt{k}}{}$$

$$\sqrt{m}$$

$$2\pi f = \frac{2\pi}{T} = \frac{\sqrt{K}}{\sqrt{m}}$$

mass

mass on spring $\boxed{T = 2\pi \frac{\sqrt{m}}{\sqrt{K}}}$ elastic constant

length

Pendulum $\boxed{T = 2\pi \sqrt{\frac{L}{g}}}$

grav. field st.

$$\omega = \frac{\sqrt{K}}{\sqrt{m}} = \frac{\sqrt{g}}{\sqrt{L}}$$

Energy of spring $= \int F\,dx = \frac{1}{2} K x^2$

$E_K = E_{total} - E_e \text{ at } x$
$= \frac{1}{2} k x_0^2 - \frac{1}{2} k x^2$

or $E_K = \frac{1}{2} m v^2$

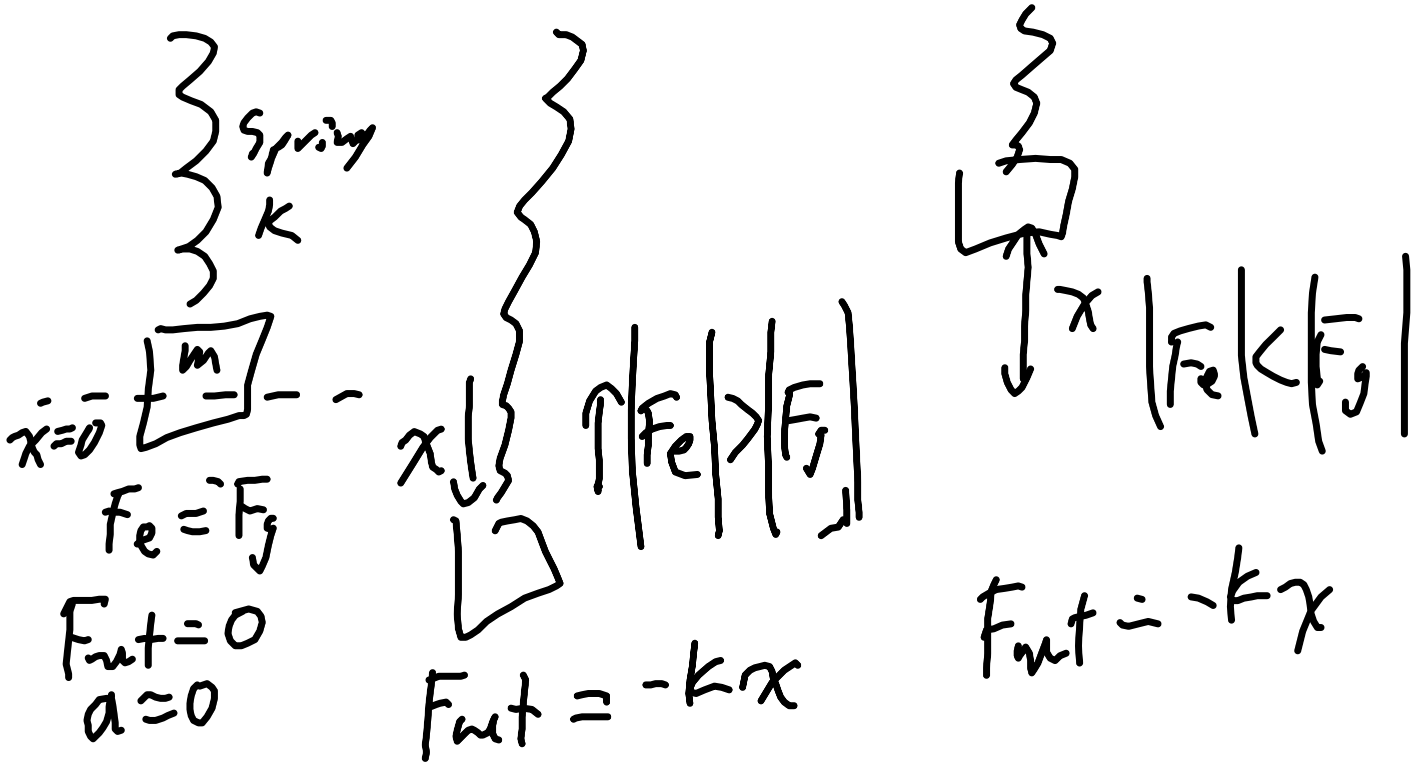
eg. a 1.0 kg mass is suspended from a spring and made to oscillate with an amplitude of 5.0 cm. If the period of oscillation is 0.90s, determine

a) the equation of x vs t - need ω,
b) k
c) graph v vs t
d) graph Ek vs t
e) the acceleration at x = 3.0 cm
f) the position of the mass after 2.30s.

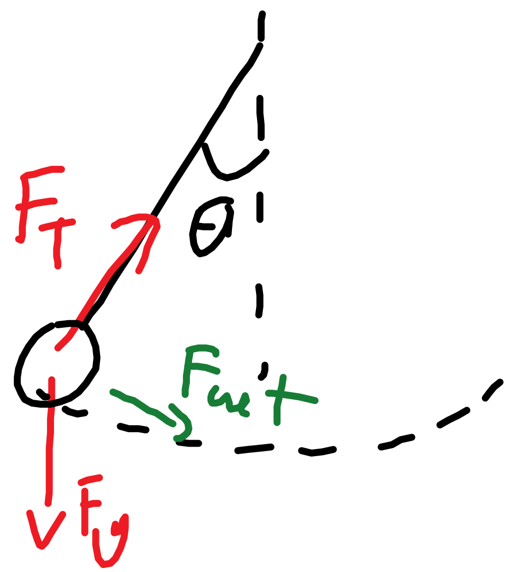
Simple Harmonic Motion, SHM
Definition:

Motion with a restoring force(towards a rest position, x=0) with the force proportional to the displacement, x.

$F_e = -kx$ for a spring



Pendulum

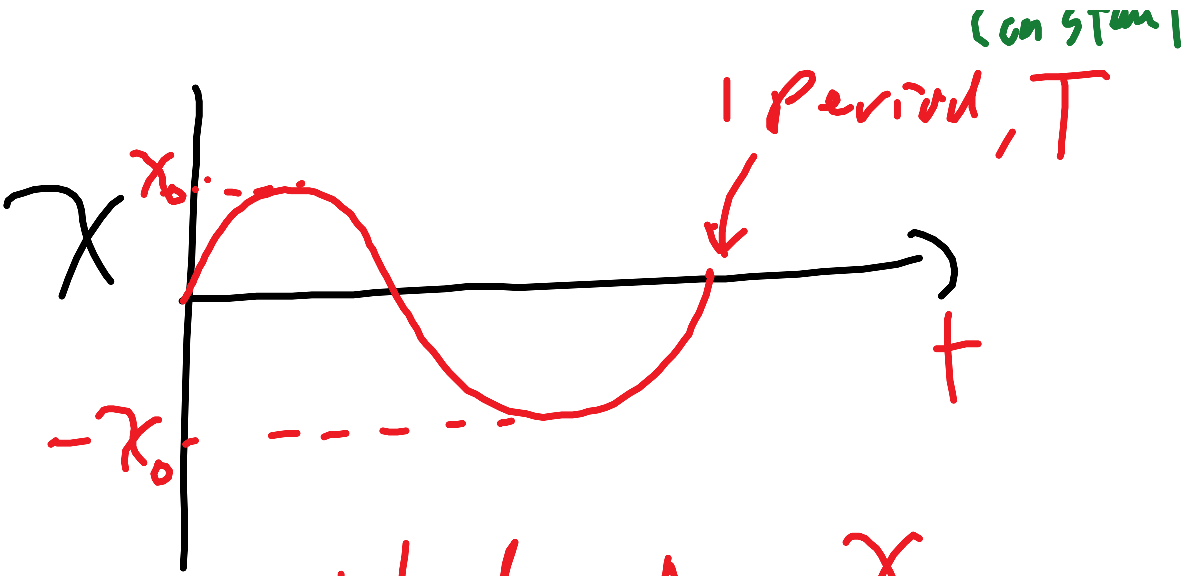


small angle approx.

$\vec{F}_{restoring} = -kx$

constant

Graph x-t



Amplitude, $A = x_0$

$$x = x_0 \sin\left(\frac{2\pi}{T} t\right)$$

in radians $\quad 360° = 2\pi =$ full rev.

angular frequency, $\omega = \frac{2\pi}{T} = 2\pi f$

units Rad/s

$$x = x_0 \sin(\omega t)$$

$$\frac{dx}{dt} = \frac{d\, x_0 \sin(\omega t)}{dt}$$

$$\frac{\ \ }{dt} \qquad \frac{\ \ }{dt}$$

calculus stuff

$$V = \omega x_0 \cos(\omega t)$$

calculus magic (chain rule)

$$V_{max} = \omega x_0$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 x_0 \sin(\omega t)$$

$$a_{max} = -\omega^2 x_0$$

from start $\rightarrow$ $ma = -kx = F_{net}$

$$m a_{max} = -k x_0$$

$$a_{max} = -\frac{k}{m} x_0$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \frac{\sqrt{k}}{\sqrt{m}} = \frac{2\pi}{T}$$

$$\boxed{T = 2\pi \frac{\sqrt{m}}{\sqrt{k}}}$$

mass

$2\pi\sqrt{\frac{m}{k}}$

elastic constant

redo for pendulum

length

$$\boxed{T = 2\pi \frac{\sqrt{L}}{\sqrt{g}}}$$

grav. field str.

$$\Delta \text{Energy} = W = \int F\,dx$$

$$E_{elastic} = \frac{1}{2} k x^2$$

$$E_k = \frac{1}{2}mv^2$$

$$E_{total} = E_e + E_k$$

$$E_{total} = E_{emax} = \frac{1}{2}Kx_0^2$$

elastic energy

$$E_K = E_t - E_e$$

$$E_K = \frac{1}{2}Kx_0^2 - \frac{1}{2}Kx^2$$

$$\frac{\sqrt{K}}{\sqrt{m}} = \omega \qquad K = \omega^2 m$$

$$\boxed{E_K = \frac{1}{2}\omega^2 m\left(x_0^2 - x^2\right)}$$

$$\boxed{V = \omega\sqrt{x_0^2 - x^2}}$$

$$\boxed{v = \omega \sqrt{x_0^2 - x^2}}$$

eg. a 1.0 kg mass is suspended from a spring and made to oscillate with an amplitude of 5.0 cm. If the period of oscillation is 0.90s, determine

a. the equation of x vs t - need ω,

$\omega = 2\pi f = 2\pi/T = 2\pi/0.90s = 6.98$ Rad/s

$A = x_o = 0.050m$

.

x= 0.050m sin(7.0 Rad/s t)

assuming t=0 at equilibrium point.

if $x = x_o$ at t=0, then

x= 0.050m cos(7.0 Rad/s t)

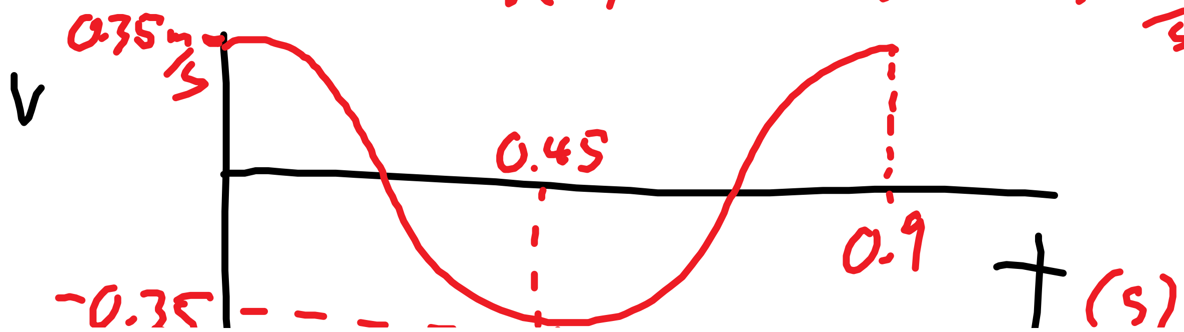
b. k

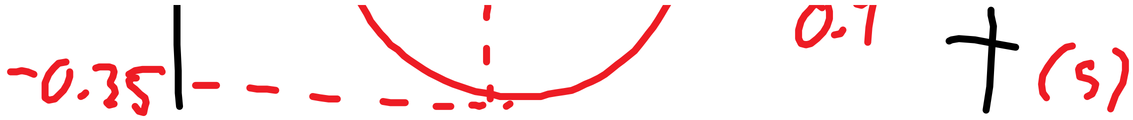
$\omega^2 = k/m$ k = (6.98 Rad/s)^2 x 1.0kg

k = 48.7 N/m (remember N=kgm/s^2, so kgm/s^2/m = kg/s^2)

=49N/m

c. graph v vs t $V_{max} = \omega x_0 = 6.98 \frac{Rad}{s} \times 0.05m$

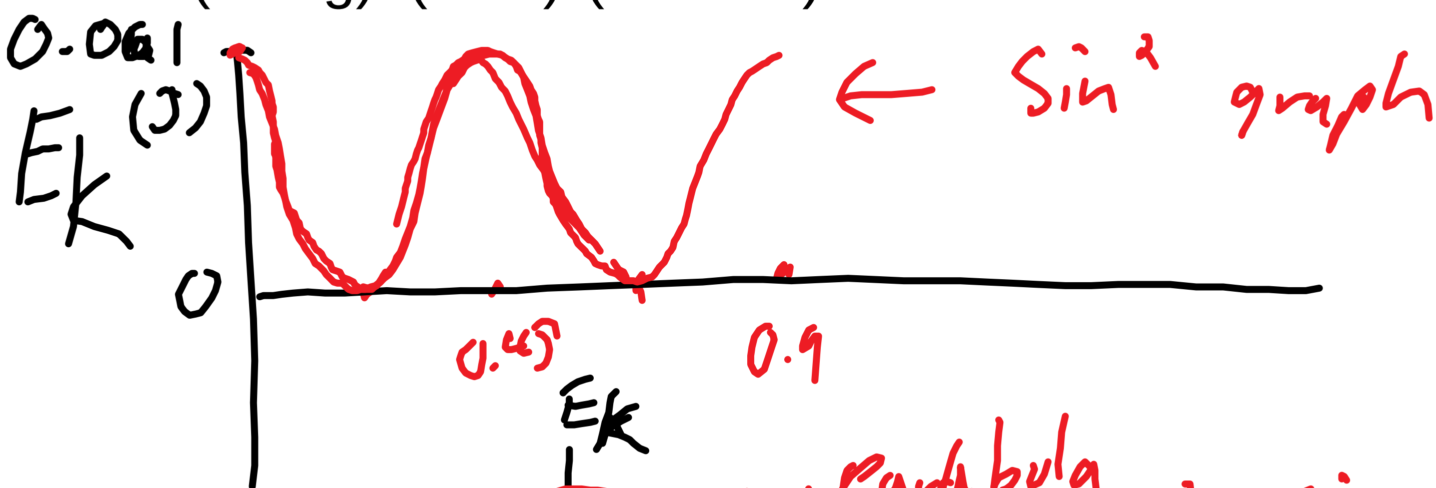


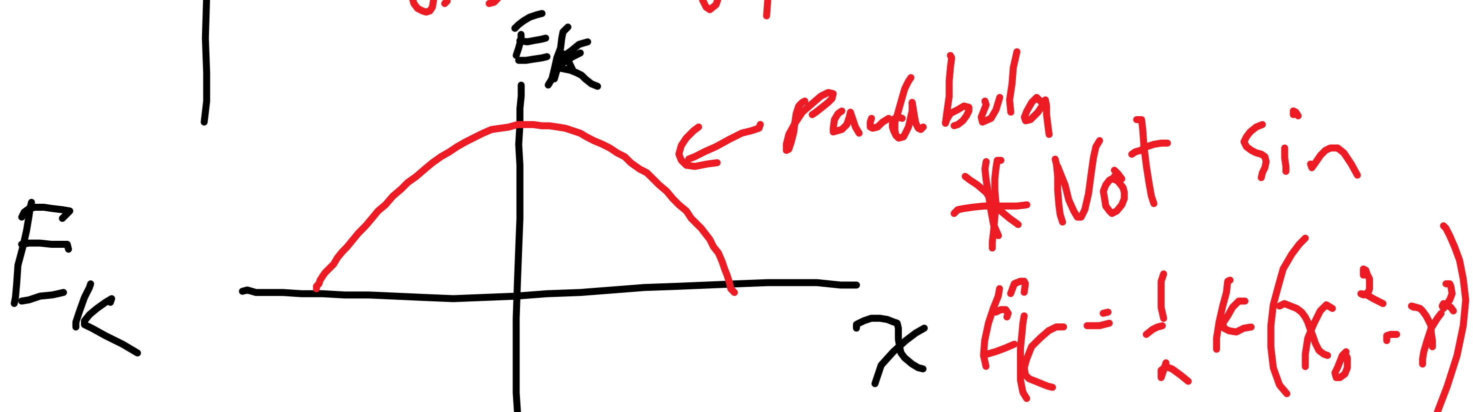


a. graph Ek vs t

Energy max = 1/2 mv^2 = 1/2 m $\omega^2 x_o^2$ =1/2 kx_o^2

=1/2 (1.0kg) $(6.98)^2(0.050\text{m})^2$= 0.061J





$$E_k = \frac{1}{2}k(x_0^2 - x^2)$$

a. the acceleration at x = 3.0 cm

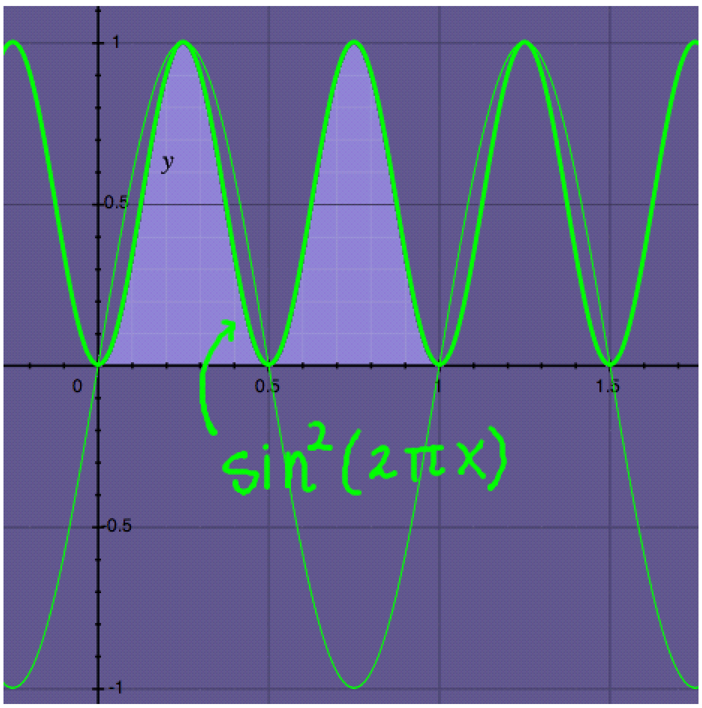
$$a = -\frac{k}{m}x \qquad a = \frac{F}{m} = \frac{-kx}{m}$$

$$= -\frac{48.7\,\text{N/m}}{1\,\text{kg}}(0.03\text{m}) = \boxed{-1.5\,\text{m/s}^2}$$

b. the position of the mass after 2.30s.

$0.05 \sin(6.98 \times 2.30\,s)$

$\boxed{= {}^{-}0.017\,m}$



$$a = \frac{F}{m} = \frac{-kx}{m} \qquad \frac{k}{m} = \omega^2$$

$$\frac{d^3x}{} = -\omega^3 x_0 \sin(\omega t) \qquad T = 2\pi \frac{\sqrt{m}}{\sqrt{k}}$$

$\frac{dx}{dt} = -\omega^2 x_0 \sin(\omega t)$

$x = x_0 \sin(\omega t)$

$T = 2\pi \frac{\sqrt{m}}{\sqrt{k}}$ $\quad \sqrt{k}$

eg. a 1.0 kg mass is suspended from a spring and made to oscillate with an amplitude of 5.0 cm. If the period of oscillation is 0.90s, determine

a) the equation of x vs t - need ω,

$\omega = 2\pi/T = 2 \times 3.14159/0.90s = 6.98$ Rad/s

$x=x_o\sin(\omega t)$ assuming x=0 at t=0

x=0.050m sin(7.0Rad/s t)

x=0.050m sin($7.0s^{-1}$ t) also acceptable

b) k

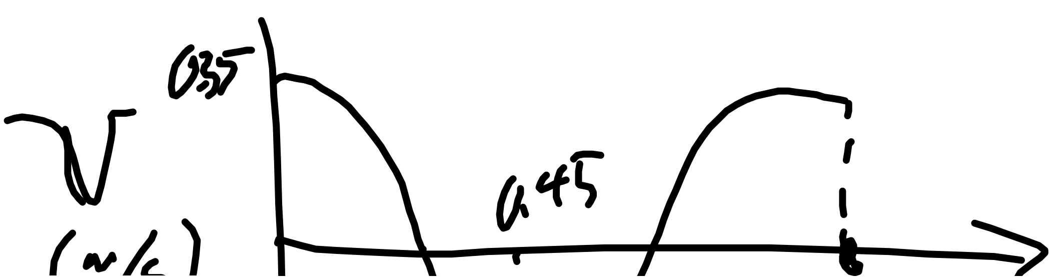
$T = 2pi\ \sqrt{m}/\sqrt{k}$

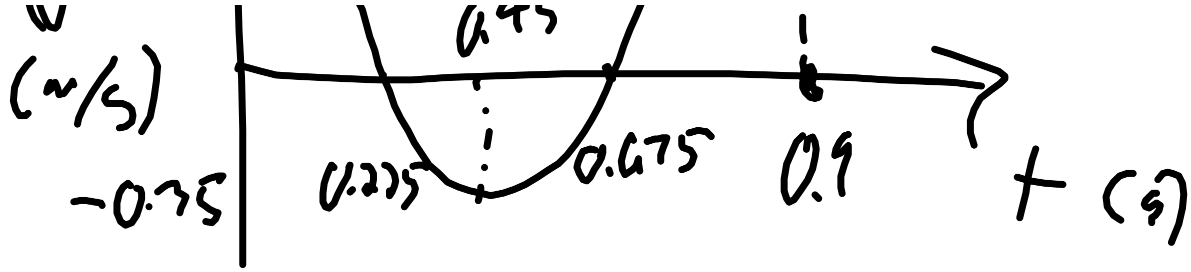
$k=\omega^2 m$

$= (6.98rad/s)^2$ 1.0 kg = 49N/m $=kg\cancel{m}/s^2\ /\cancel{m}$

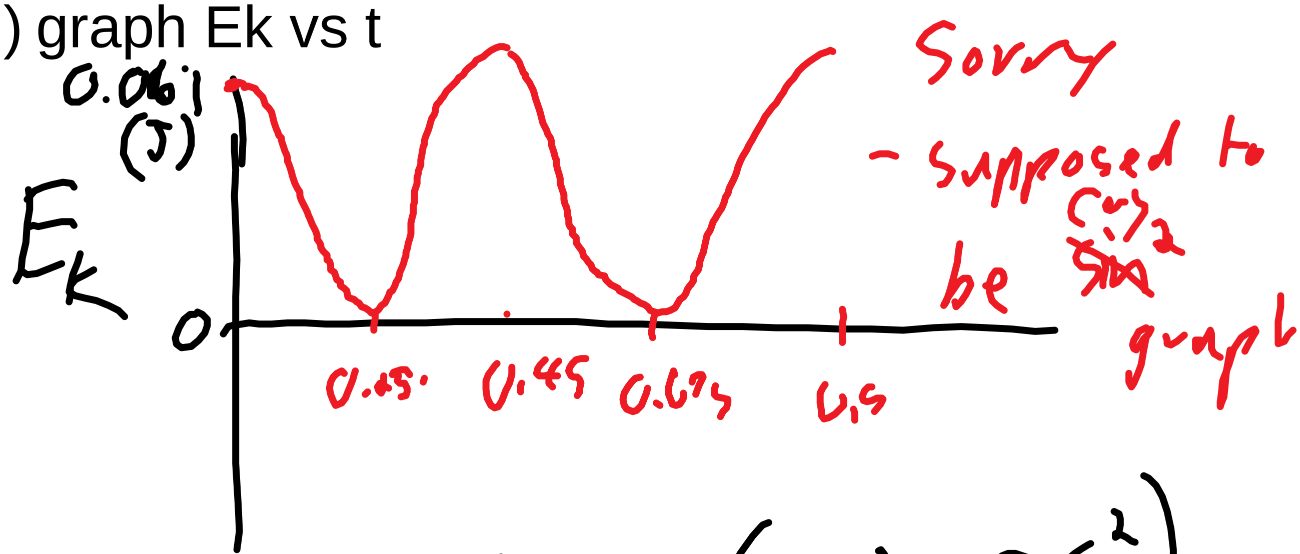
c) graph v vs t

$v_{max} = \omega x_o = 6.98 \times 0.05 = 0.35$ m/s

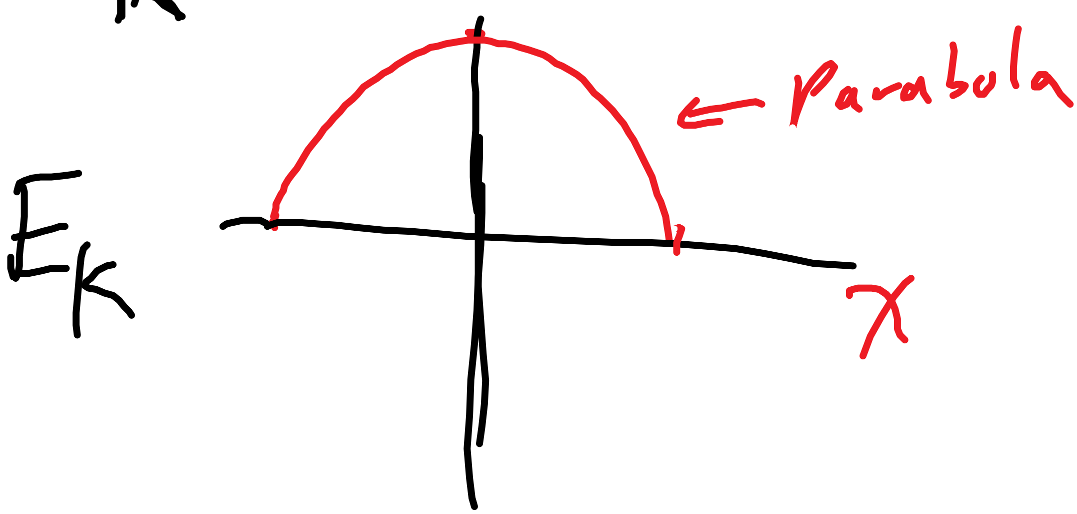




d) graph Ek vs t



$$E_k = \frac{1}{2}k\left(x_0^2 - x^2\right)$$



a) the acceleration at x = 3.0 cm

a=F/m = -kx/m =$-\omega^2$x = -(6.98)^2 0.05 = - 2.4m/s^2

b) the position of the mass after 2.30s.

$x = 0.050\text{m} \sin(7.0\text{Rad/s } t)$
$x = 0.050\text{m} \sin(7.0\text{Rad/s } 2.30\text{s})$
$x = -0.019\text{m}$

SHM

definition: motion with a force proportional to the displacement and opposite in direction.

eg. a pendulum has length 1.65m and a bob mass of 1.24 kg. If you pull the bob 10.0 cm to the side and release set t=0, (x is the horizontal displacement of the bob)

a) what is the period of the pendulum?

= 2pi root(l/g) = 2 pi root(1.65m/9.81m/s^2)
2.58s

b) what is the angular frequency?

ω= root(g/l) = sqrt(9.81/1.65)=2.438330278172862 2.44 rad/s
or 2 pi/T = 2xpi/2.58 = 2.435343142317669
2.44 rad/s

c) what are the equations for x, v, a vs t?

$\boxed{x = 0.10\text{m} \cos(2.44 \text{ rad/s } t)}$

$v = -\omega x_o \sin(2.44 \text{ rad/s } t)$

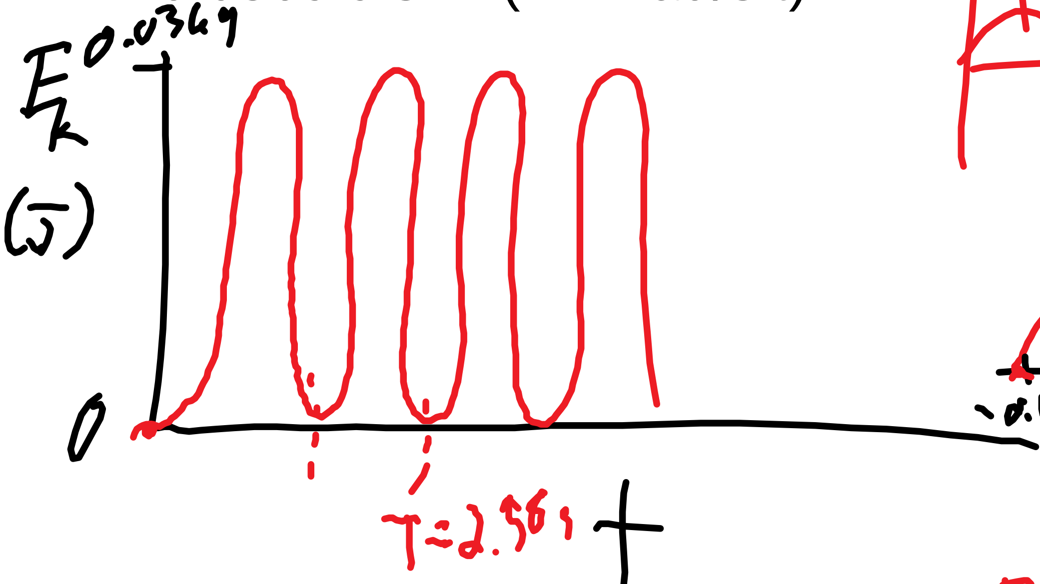
$\boxed{v = -0.244\text{m/s} \sin(2.44 \text{ rad/s } t)}$

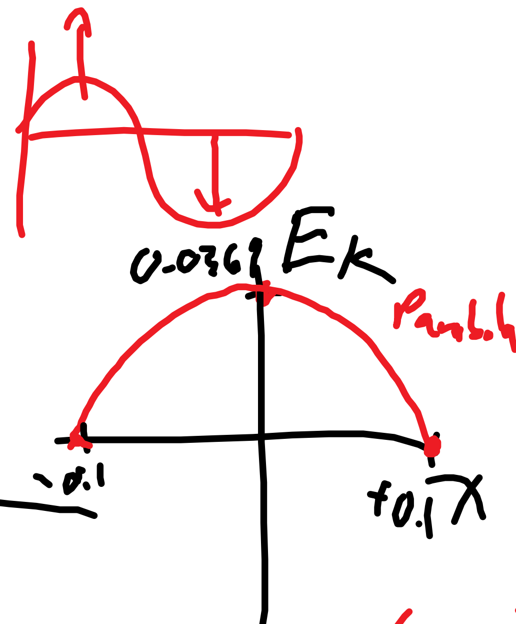
$a = -\omega^2 x = -2.44^2\, 0.1 \cos(2.44\text{rad/s } t)$

$\boxed{a = -0.594 \text{ m/s}^2 \cos(2.44 \text{ rad/s } t)}$

d) sketch Ek vs t and vs x

Ek = 1/2 mv^2 = 1/2 (1.24kg) (-0.244m/s sin (2.44 rad/s t)$)^2$
= 0.5x1.24x0.244^2=0.0369 J
Ek = 0.0369 J $\sin^2$ (2.44rad/s t)





$$E_K = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$$

$$E_K = \frac{1}{2} m v^2$$

a) sketch v vs x

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

$$\left(\frac{v}{\omega}\right)^2 = x_0^2 - x^2$$

$$\frac{y^2}{c} = A^2 - x^2$$

$$y^2 = 1 - x^2$$

b) what is the speed at x = 7.2 cm?

v = 2.44xsqrt(0.1^2-0.072^2)=0.169329671351479
17 cm/s or 0.17 m/s

kinematics review:

1. A car is moving at a constant 140 km/h past a police car at rest. 2.0 s later, the police car accelerates at 7.5m/s^2.

a) how fast is the speeding car in m/s?
140km/h(1000m/km)(h/3600s) =
140/3.6=38.8889 m/s = 39m/s

b) how far did the car go in the 2.0s?
38.8889x2=77.7778 =78m d=vt

c) How long will it take for the police car to reach the same speed as the car?
vi, vf, a, t
v=at+u t= 38.8889/7.5=5.1852 =5.2s

d) How long(from when the car passes the police) before the police car catches the car?

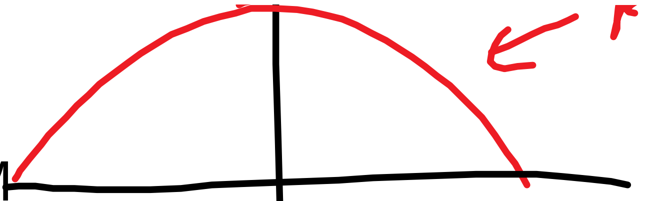
e) where does the police car catch the car?

2. You throw a ball up in the air at 3.0 m/s from a height of 1.5m.

a) how high does it go?

b) what is the speed after 1.5s?

c) when does it hit the ground?



SHM

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eg. a pendulum has length 1.65m and a bob mass of 1.24 kg. If you pull the bob 10.0 cm to the side and release at t=0, (x is the horizontal displacement of the bob)

$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(x_0^2 - x^2)$

a) what is the period of the pendulum?

v 0.244 m/s

$t = 2pi\frac{\sqrt{L}}{\sqrt{g}}$

2x3.14159x1.284/3.132=2.57586

= 2.5768 s = 2.58s

$v = \omega\sqrt{x_0^2 - x^2}$

x

0

b) what is the angular frequency?

$y = \sqrt{1 - x^2}$

ω = 2pi/T = 2x3.14159/2.5768=2.43837

$x_0 = 0.1m$

= root(g/L)

2.44 Rad/s

c) what are the equations for x, v, a vs t?

x = 0.10m cos (2.44Rad/s t)

v= $-\omega x_o \sin(\omega t)$ = - 0.244m/s sin(2.44Rad/s t)

a=$-\omega^2$ x = $-(2.44s^{-1})^2$ 0.10m cos (2.44Rad/s t)

a= 2.44x2.44=5.9536

a=-0.595 m/s^2 cos (2.44Rad/s t)

d) sketch Ek vs t and vs x

$E_k = 1/2\ mv^2 = 1/2\ m\ (-\omega x_o \sin(\omega t))^2$

$1/2\ m\ \omega^2 x_o^2 \sin^2(\omega t)$

$= 1/2\ (1.24kg)\ (2.44)^2 (0.1)^2 \sin^2\ (2.44rad/s\ t)$

0.5x1.24x2.44x2.44x0.01=0.0369 J

a) sketch v vs x

-0.1

a) what is the speed at x = 7.2 cm?

0.189 m/s

kinematics review:

1. A car is moving at a constant 140 km/h past a police car at rest. 2.0 s later, the police car accelerates at $7.5 m/s^2$.
a) how fast is the speeding car in m/s?
b) how far did the car go in the 2.0s?
c) How long will it take for the police car to reach the same speed as the car?
d) How long(from when the car passes the police) before the police car catches the car?
e) where does the police car catch the car?
1. You throw a ball up in the air at 3.0 m/s from a height of 1.5m.
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b) what is the speed after 1.5s?
c) when does it hit the ground?

Thursday, September 24, 2015
12:36 PM

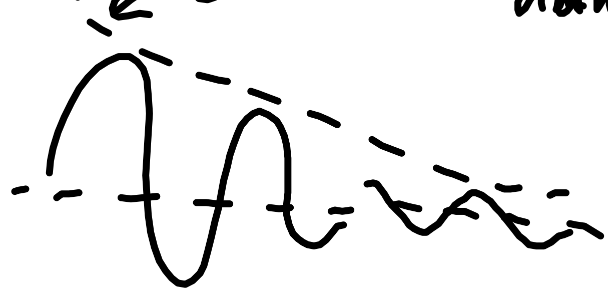
Quiz
review kinematics
Giancoli p37 Q5-17 odds
p38-41 problems 7, 19, 27, 33, 41, 43

Kinematics
"things move" Graham

$$x = x_0 e^{-\lambda t} \sin\omega t$$

damping

$e^{-\lambda t}$



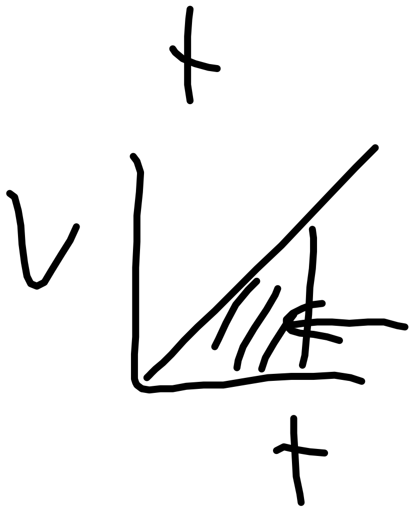
Constant v

$\cancel{d = vt}$

$s = vt$

Constant a

$$s = \frac{1}{2}at^2 + ut$$



$v = at + u$

area $= s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

if air resistance is negligible,
then $a = g = 9.81\ m/s^2$

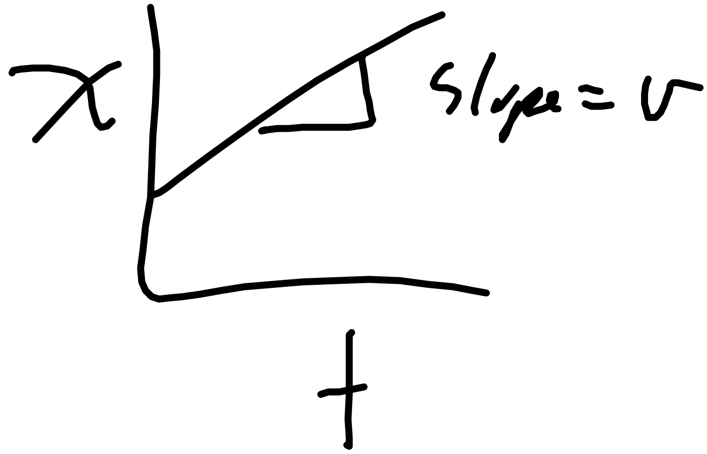
Near Earth

2-4
Quiz
review kinematics
Giancoli p37 Q5-17 odds
p38-41 problems 7, 19, 27, 33, 41, 43

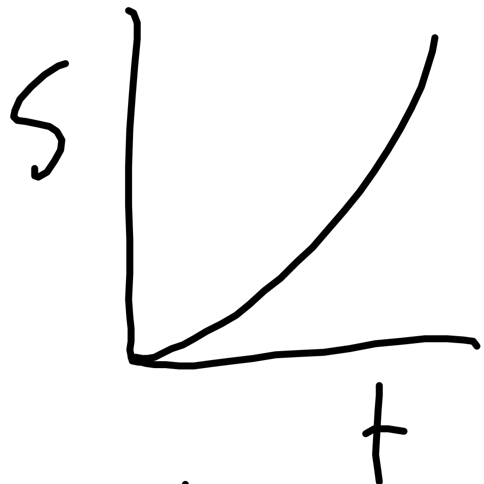
Kinematics
"things move" Nicholas

Constant v

$s = vt$

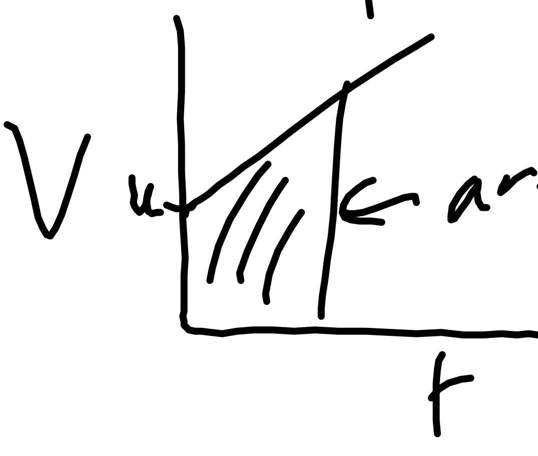


Constant a



$s = \frac{1}{2}at^2 + ut$

$v = at + u$



area $= s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

if air resistance is negligible

a of a falling object = g

$= 9.81\,m/s^2$

Near Earth

1. A car is moving at a constant 140 km/h past a police car at rest. 2.0 s later, the police car accelerates at 7.5m/s^2.

a) how fast is the speeding car in m/s?
140km/h(1000m/km)(h/3600s) =
140/3.6=38.8889 m/s = 39m/s

b) how far did the car go in the 2.0s?
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c) How long will it take for the police car to reach the same speed as the car?
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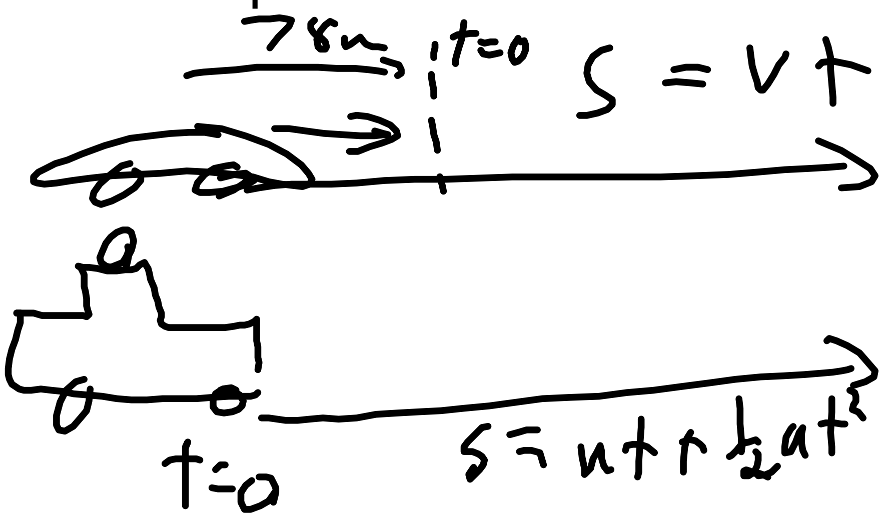
d) How long(from when the car passes the police) before the police car catches the car?



$s = vt$

$ax^2+bx+c=0$

$78+vt=ut+\frac{1}{2}at^2$

$78+39t=3.75t^2$

$s = ut + \frac{1}{2}at^2$

$$\frac{-39 \pm \sqrt{39^2-4(-3.75)78}}{2(-3.75)}$$

$t = 12.1s + 2s = \boxed{14s}$

a) where does the police car catch the car?
s=vt= 38.89x14.11=548.7379 =5.5x10^{2}m

$s=1/2at^2 = 1/2(7.5)(12.11)^2=$
0.5x7.5x12.11^2=549.9454
0.54km from where the car passes the police car

1. You throw a ball up in the air at 3.0 m/s from a height of 1.5m.

a) how high does it go?
$v^2=u^2+2ad$ 3^2/(2x9.81)=0.4587 m
0.46m above the launch point or 2.0 m above the floor.

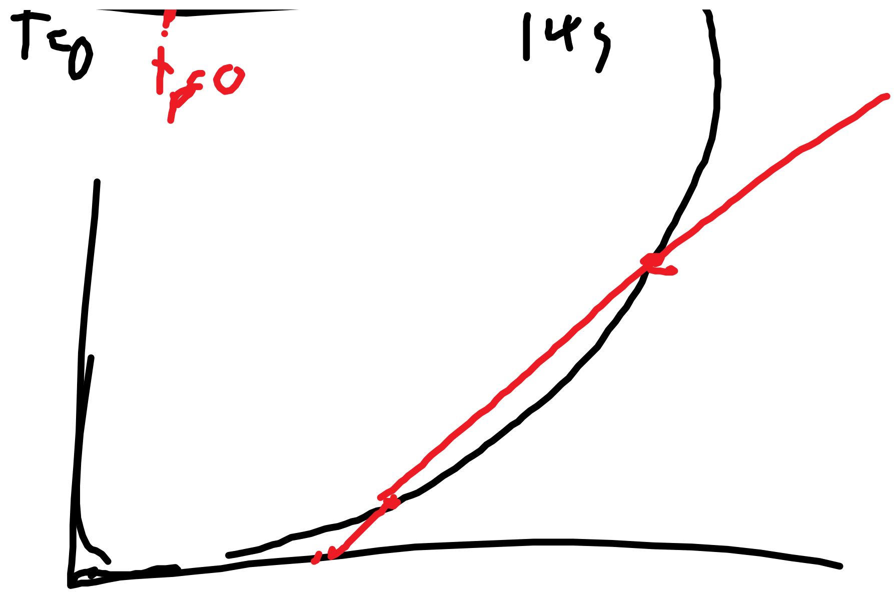
b) what is the speed after 1.5s?
v=at+u = -9.81x1.5+3=-11.715 = -12m/s

c) when does it hit the ground?
$d=1/2at^2$ t= sqrt(2x1.9587/9.81)=0.631923426631251
v=at+u t= (0-3)/-9.81=0.3058
total = 0.94s

$d=1/2at^2+ut$
$-1.5=1/2(-9.81)t^2 + 3t$
$0 = -4.9t^2 +3t +1.5$

$$\frac{-3 \pm \sqrt{9-4(-4.9)(1.5}}{-9.8}$$

$$\boxed{t=0.94s}$$



Vectors in 2D

What quantities are

vectors?	scalars?
v,a,s,F,g=F/m	W, m, V, Energy, Luminosity, Power,
I, t?,	Resistance, speed, length, f, t

Adding and subtracting vectors
(multiplying we will look at with
Work/torque/magnetism)

Basics
diagram: you draw vectors as arrows,

adding, draw the vectors head to tail
resultant is the vector from the tail of the first vector to the head of the last vector.

subtracting or the change in a vector
just flip the direction of the second vector.

3 solution methods:

1. scale diagram with ruler and protractor
2. sine/cosine law for 2 vectors
3. vector components - any vector can be represented as either a magnitude and direction (polar coordinates) or x and y components (Cartesian graph)

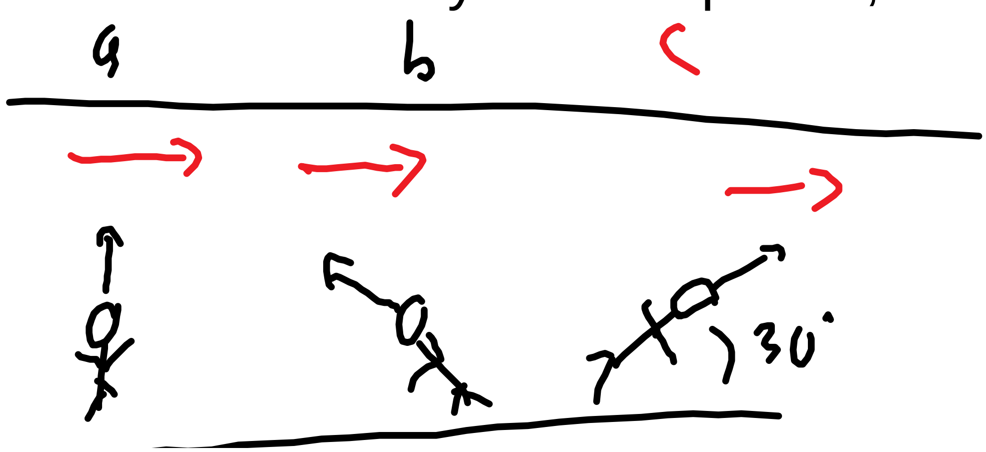
eg. You swim at 2.00m/s across a 50.0m wide river flowing at 1.20m/s parallel to the shore.

i) Determine your resultant velocity if you point

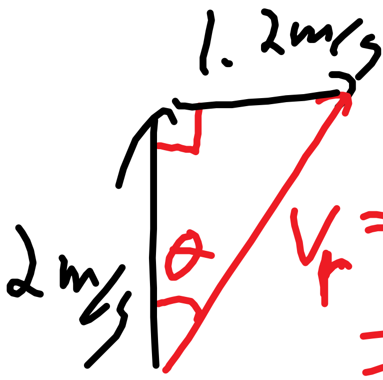
a) straight across b) angle so you end up straight across c) you swim downstream at 30.0º to the shore

ii) determine the time to cross for a,b and c

iii) determine where you end up for a,b and c



a)

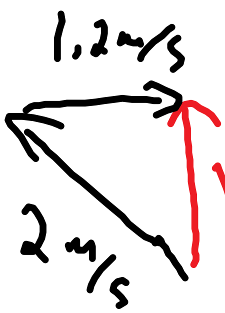


$V_r = \sqrt{1.2^2 + 2^2}$

$= \boxed{2.3\ m/s}$

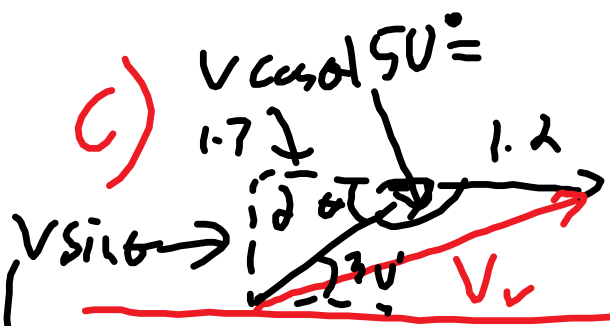
$\theta = \tan^{-1}\frac{1.2}{2} = \boxed{31^\circ \text{ downstream}}$

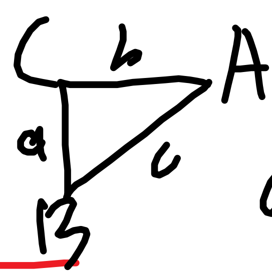
b)



$V_r = \sqrt{2^2 - 1.2^2} = \boxed{1.6\ m/s \text{ directly across}}$

c)





$c^2 = a^2 + b^2 - 2ab\cos C$

$2\sin 30 = 1.$

$V_r^2 = 2^2 + 1.2^2 - 2(2)(1.2)\cos 150$

$= 3.1\ m/s$

$$V_r = \sqrt{2.9^2 + 1^2} = 3.1\,m/s$$

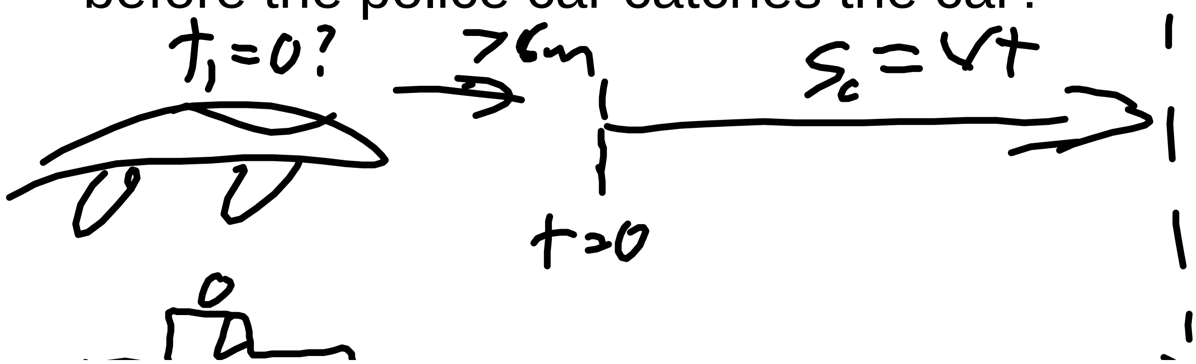
1. A car is moving at a constant 140 km/h past a police car at rest. 2.0 s later, the police car accelerates at 7.5m/s^2.

a) how fast is the speeding car in m/s?
140km/h(1000m/km)(h/3600s) =
140/3.6=38.8889 m/s = 39m/s

b) how far did the car go in the 2.0s?
38.8889x2=77.7778 =78m s=vt

c) How long will it take for the police car to reach the same speed as the car?
u, v, a, t
v=at+u t= 38.8889/7.5=5.1852 =5.2s

d) How long(from when the car passes the police) before the police car catches the car?



$t_2 = 0$ $\qquad s_p = \frac{1}{2}at^2$

$$\frac{1}{2}at^2 = Vt + 78$$

$$3.75t^2 = 39t + 78$$

$$0 = -3.75t^2 + 39t + 78$$

$$\frac{-39 \pm \sqrt{39^2 - 4(-3.75)(78)}}{-7.5} = 12.1$$

$$t = t_p + 2s = 12.1s + 2s = \boxed{14.1s}$$

a) where does the police car catch the car?
   s=vt = 39x14.1=549.9
   s=1/2at^2 = 0.5x7.5x12.1^2=549.0375

1. You throw a ball up in the air at 3.0 m/s from a height of 1.5m.

a) how high does it go?
   u=3.0m/s v=0 a=g=-9.81m/s^2 s=?

$v^2=u^2+2as$
$s=-u^2/2a$ = 3^2/(2x9.81)=0.4587
0.46m above launch point or
2.0m above the ground.

b) what is the speed after 1.5s?
v=at+u
v= -9.81x1.5+3=-11.715 =-12m/s

-12m/s

c) when does it hit the ground?

$s=1/2at^2+ut$
$-1.5 = 1/2(-9.81)t^2 + 3t$
$0 = -4.905t^2 + 3t + 1.5$
(-3-(sqrt(9-((4)(-4.905)(1.5)))/(-9.81)))=
0.94s

tup = 3/9.81=0.3058
tdown = sqrt(2x1.9587/9.81)=0.631923426631251
t= 0.3058+0.6319=0.9377 =0.94s

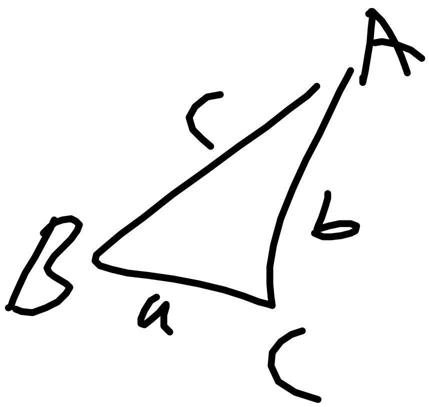
Vectors:
quantities with magnitude and direction

vectors:	scalars
s,v,a,F,p, current (magnetism), time rel.	m, speed, length, Power, Energy, $, current, time, Temp,

Adding vectors: draw them head to tail.
subtracting or the change in a vector: add vectors but reverse the direction of the second vector.

actual solutions can be found using 3 methods:

1. scale vector diagram with a ruler and protractor
2. sine/cosine law



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

3. components - vectors can be represented in 2 ways,

a) magnitude and direction (polar coordinates)
b) as the sum of 2 perpendicular vectors call components (Cartesian coordinates)

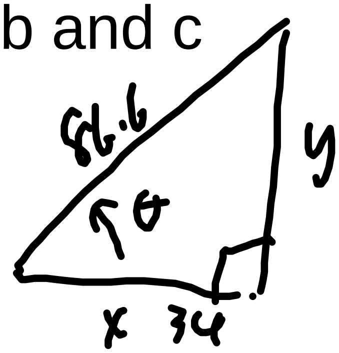
eg. You swim at 2.00m/s across a 50.0m wide river flowing at 1.20m/s parallel to the shore.

i) Determine your resultant velocity if you point

a) straight across b) angle so you end up straight across c) you swim downstream at 30.0º to the shore

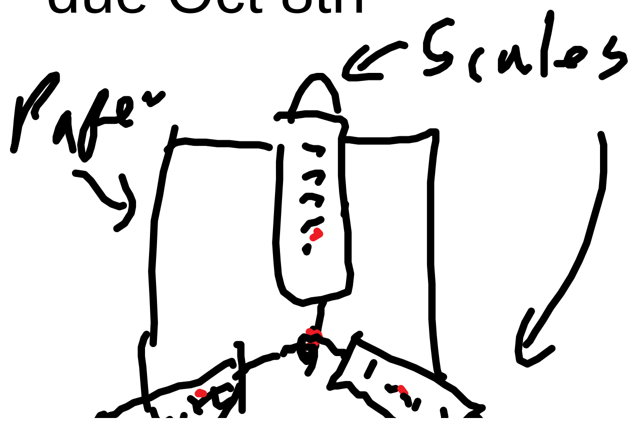
ii) determine the time to cross for a,b and c

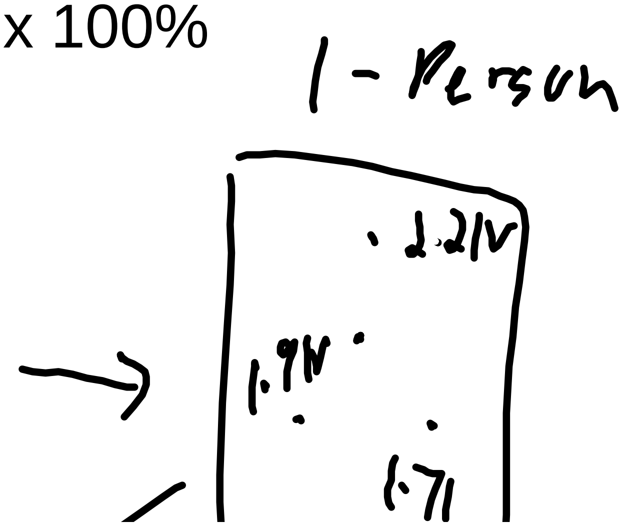
iii) determine where you end up for a,b and c

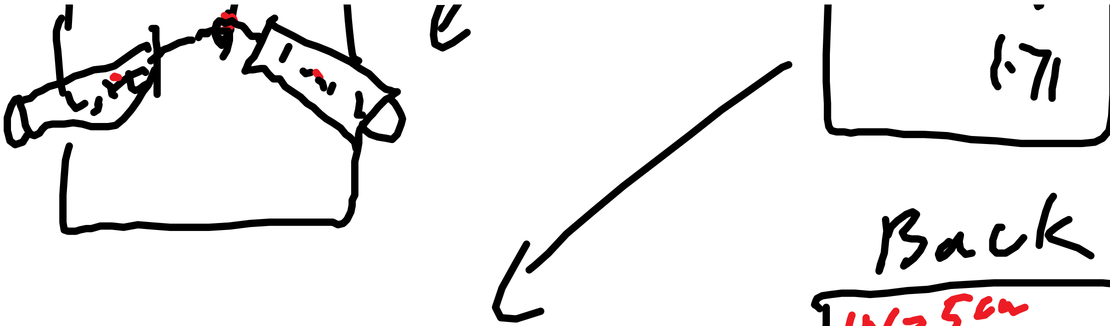


Free Body Diagram:
3 force scales on a board
record each force and direction
draw a scale diagram of the forces.
draw a scale diagram of the vector addition of the 3 forces
calculate the resultant force from

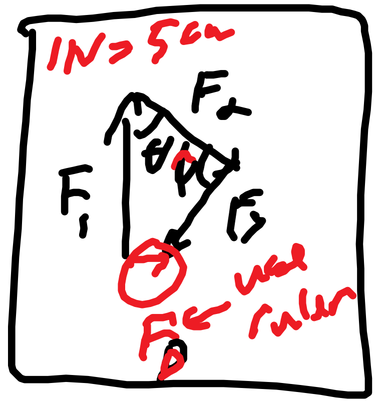
1) scale diagram 2) components
%deviation = $(F_d - F_c)/F_c \times 100\%$
due Oct 8th

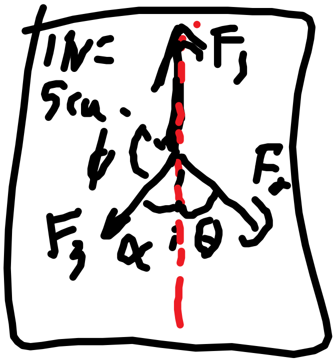






Back





Free body diagram

Vector Addition

$$F_{y_1} - F_2\cos\theta - F_3\cos\alpha = \sum F_y$$

$$\cancel{F_{y_1}}^{0} + F_2\sin\theta - F_3\sin\alpha = \sum F_x$$

$$F_c = \sqrt{\sum F_y^2 + \sum F_x^2}$$

$$\%\,\text{deviation} = \frac{F_D - F_c}{F_c} \times 100\%$$

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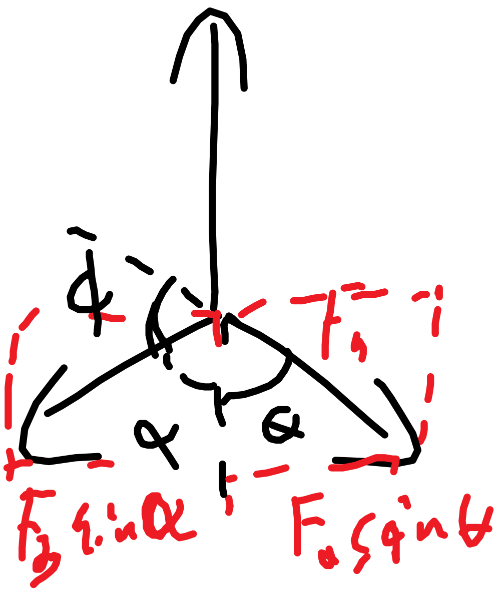
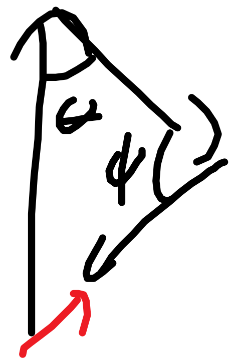
Lab:





ΣF_x $\quad F_c = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

ΣF_y

Projectiles

2Demos:

1. drop hammer, feather, paperclip
   hammer and paperclip land at the same time
   because air resistance is negligible.
   $a=g=-9.81 m/s^2$

2. drop a pen while I throw a pencil horizontally.
   The both hit at the same time.
   vertical and horizontal motion is independent.
   (mythbusters, dropped and fired bullets video)

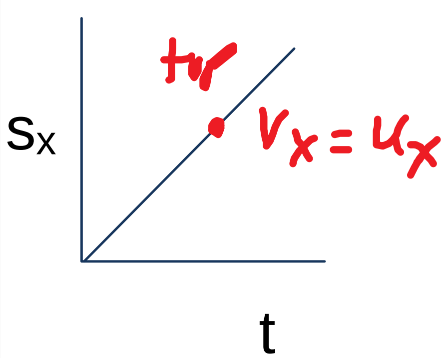
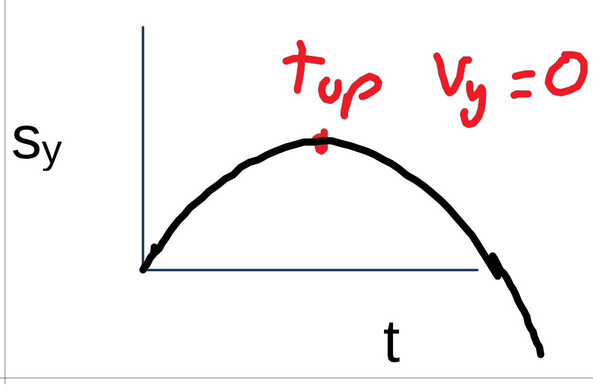
https://www.youtube.com/watch?v=abUBrQmI33Q

http://techtv.mit.edu/videos/735-monkey-and-a-gun

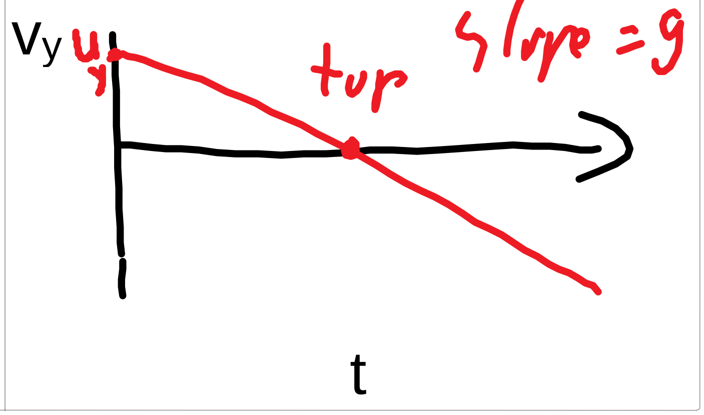
Procedure for solving projectile problems:

1. separate the velocity into horizontal, x, and vertical, y, components.
2. draw a diagram including the givens and unknowns.
3. find equation relating unknowns to knowns. (often you have to find time first, then distance)

equations:
assuming air resistance is negligible,
horizontal motion is constant velocity.
vertical motion is constant acceleration.

x	y
$s_x = v_x t$	$s_y = ½gt^2 + u_y t$
	

$v_y = gt + u_y$



eg. You run off a 15.0 m high cliff and land in water. If your speed is 7.00m/s what is

i) time to land ii) landing point iii) speed and direction just before you hit the water

if

a) you run horizontally off the cliff

b) you jump at the edge up so you have the same speed but up at 37.0º above the horizontal.

a) $d = \frac{1}{2}at^2 + \cancel{ut}^{\to 0}$

$t = \sqrt{\frac{2(-15)}{-9.81}} = 1.75\,s$

b) $S_x = V_x t = 7\,m/s\,(1.75\,s)$

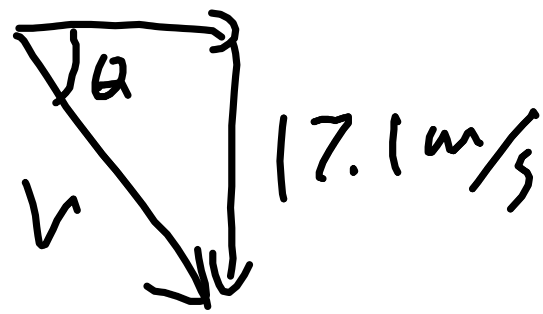
$= \boxed{12.3\,m}$

$$= \boxed{12.3\text{m}}$$

c) $$v_y^2 = \cancel{u_y^2}^{\to 0} + 2gs_y$$

$$v_y = \sqrt{2(-9.81)(-15)} = 17.1\text{m/s}$$

$v_x = 7\text{m/s}$



$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{7^2 + 17.1^2}$$

$$= \boxed{\begin{array}{l} 18.5\text{m/s } 67.7^\circ \\ \text{below horizontal} \end{array}}$$

Projectiles
define: an object moving through the air without lift or propulsion.

2 demos:

1. hammer, feather, paperclip all fall. feather has air resistance, so it falls slower but the hammer and paper clip have negligible air resistance so they fall at $9.81 m/s^2$.

2. drop a pen while throwing a pencil sideways. They land at the same time. Therefore perpendicular vectors are independent. (exceptions: fluid flow and relativistic dynamics)

Projectile problems:

1. Separate vectors into horizontal, x, and vertical, y, components.
2. Draw diagram with givens and unknowns.
3. Find equations relating knowns and unknowns. common to have to find time them distance.

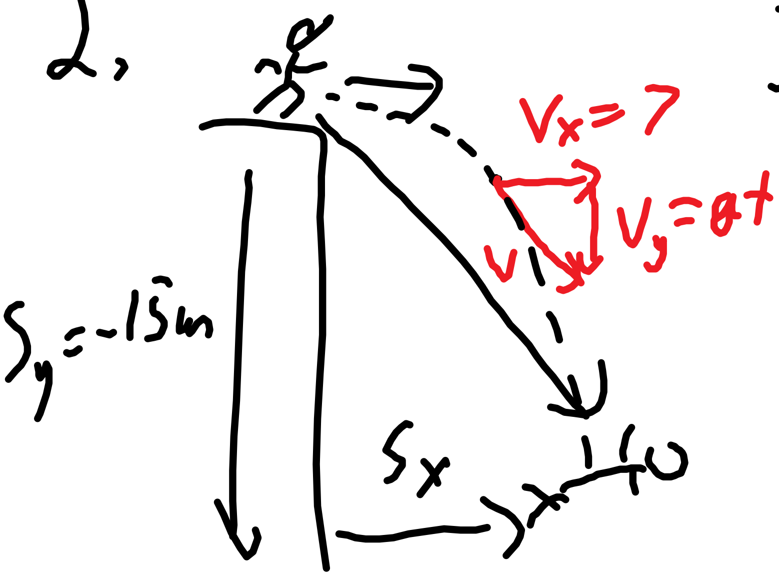
assuming air resistance is negligible:
Only force on the projectile is gravity.
horizontal motion is constant velocity, v_x
vertical motion is constant acceleration,

$g=-9.81 m/s^2$

x	y

1. $V_x = 7.00 m/s \quad u_y = 0$

2.



3) $S_y = \frac{1}{2}gt^2 + \cancel{ut}^{\,0}$

$$t = \sqrt{\frac{2(-15m)}{-9.81 m/s^2}}$$

$$= \boxed{1.75 s}$$

$$S_x = V_x t = 7.0 m/s (1.75 s)$$

$$= \boxed{12.3 m}$$ from base of cliff

iii) $V_y^2 = u_y^2 + 2gs$

$V_y = \sqrt{2(-15)(-9.8)} = 17.1 \text{ m/s}$

$V_x = 7 \text{ m/s}$

$V = \sqrt{17.1^2 + 7^2} = \boxed{18.5 \text{ m/s}}$

$\theta = \tan^{-1} \frac{17.1}{7} = \boxed{67.8^\circ \text{ below horizontal}}$

zd dfas

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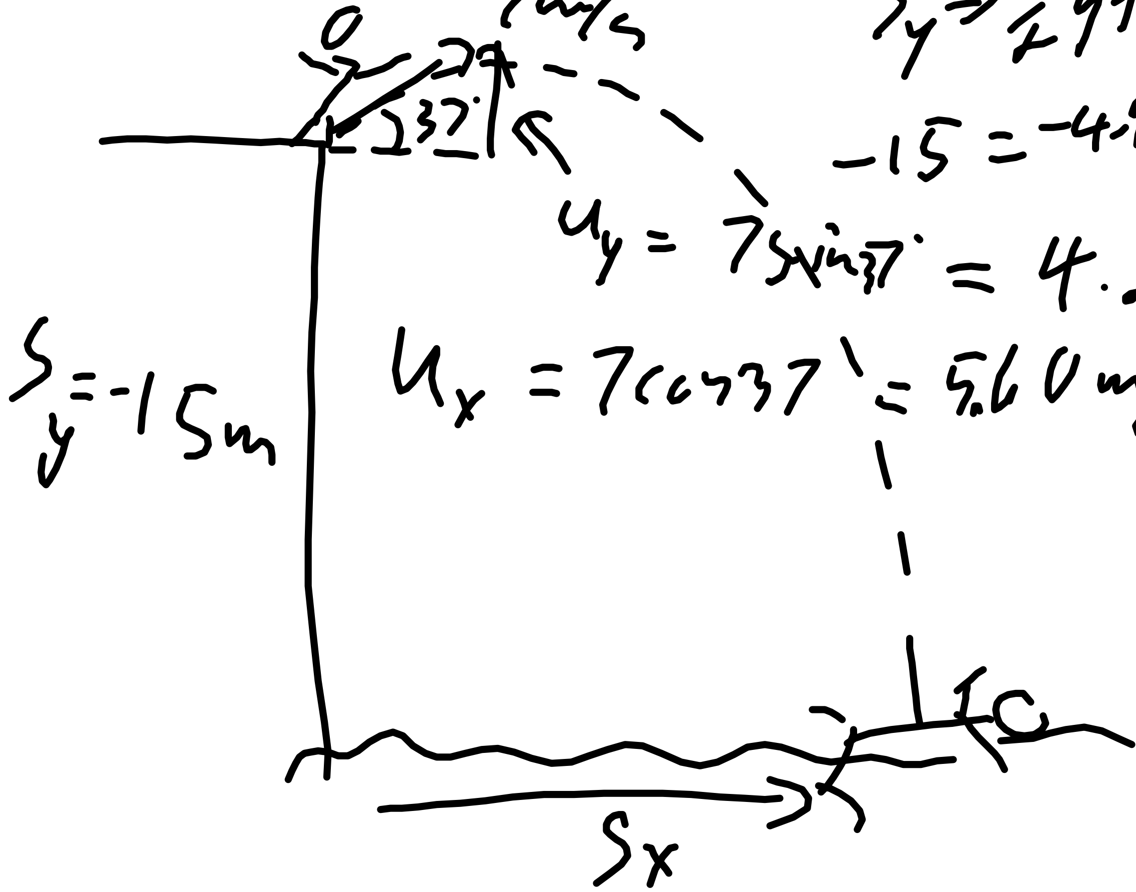


$$S_y = \frac{1}{2} g t^2 + u_y t$$

$$-15 = -4.9t^2 + 7\sin 37\, t$$

$$u_y = 7\sin 37^\circ = 4.21 \text{ m/s}$$

$$u_x = 7\cos 37^\circ = 5.60 \text{ m/s}$$

$$-4.9t^2 + 4.21t + 15 = 0$$

$$\frac{-4.21 \pm \sqrt{4.21^2 - 4(-4.9)(15)}}{-9.8}$$

$$\frac{-4.21 \pm 17.656}{-9.8} = \boxed{2.23 \text{ s}}$$

$$S_x = V_x t = 5.60 \times 2.23 = \boxed{12.5 \text{ m}}$$

$\underline{V\cos\theta}$

$V_x = 5.60 \text{ m/s}$

$V_y^2 = V_{iy}^2 + 2 g d_y$

$V_y^2 = (4.11)^2 + 2(-9.8)(-15)$

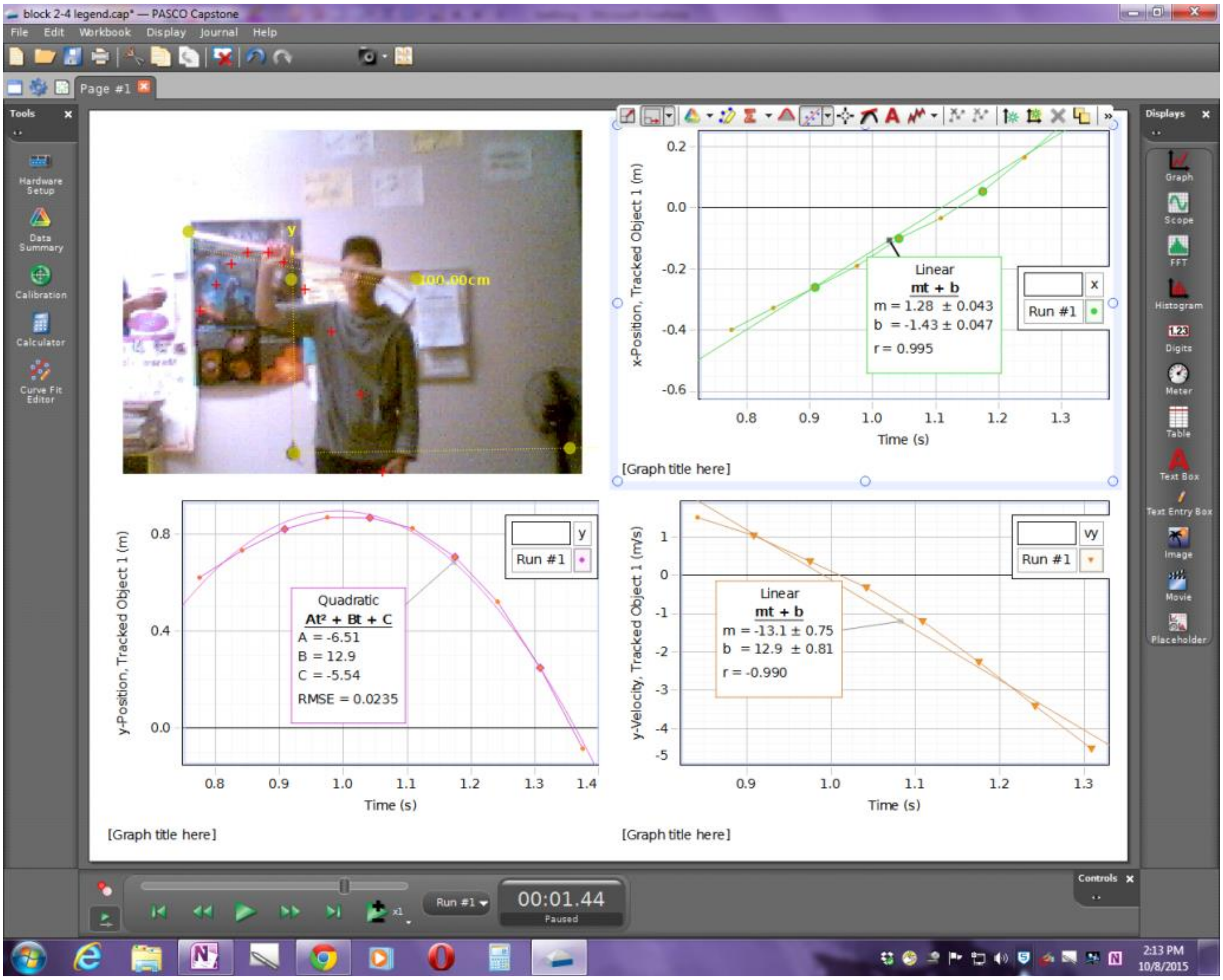
$V_y = 17.66 \text{ m/s}$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{5.6^2 + 17.66^2}$

$= \boxed{18.5 \text{ m/s}}$

$\theta = \tan^{-1} \frac{17.66}{5.6}$

$= \boxed{72^\circ \text{ Below horizontal}}$



s_x = 1.28+/-0.04m/s t -1.43+/-0.05m
sy = -6.51m/s^{2}t^2 + 12.9 m/s t -5.54m

experimental value, exp = slope of vy-t graph

%deviation=|exp-theo|/theo x100% =

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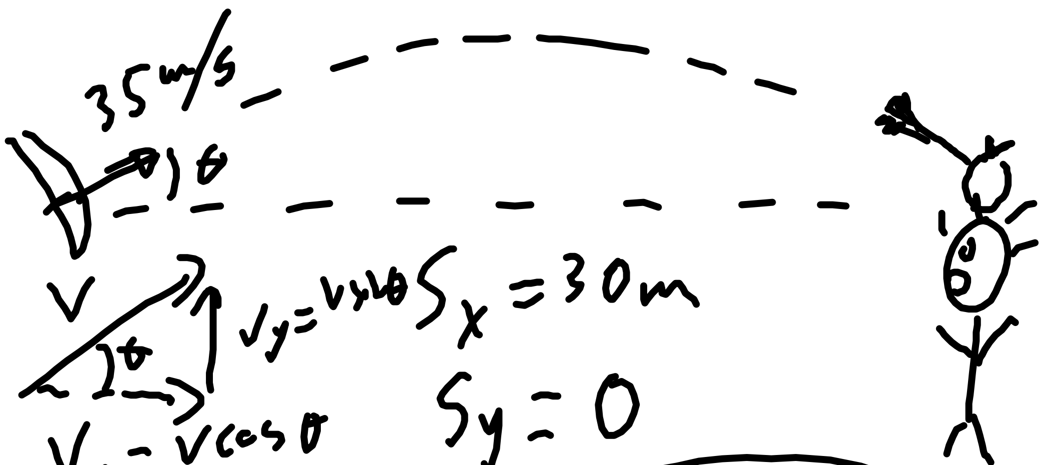
Projectiles: HW
45



$V_y = V\sin\theta$ $\quad S_x = 30\,m$

$V_x = V\cos\theta$ $\quad S_y = 0$

$$S_x = V_x t \qquad S_y = \frac{1}{2} g t^2 + V_y t$$

$$S_y = \frac{1}{2} g \left(\frac{S_x}{V_x}\right)^2 + V_y \left(\frac{S_x}{V_x}\right)$$

$$S_y = \frac{1}{2} g \frac{S_x^2}{V^2\cos^2\theta} + \frac{V\sin\theta \quad S_x}{V\cos\theta}$$

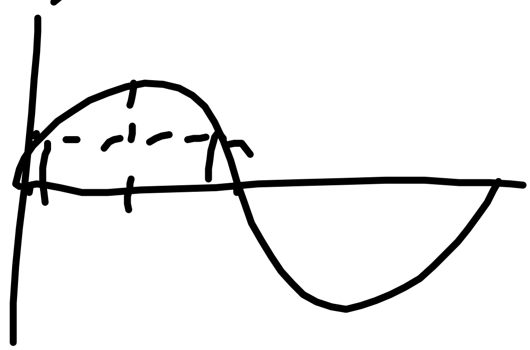
$$\cancel{S_y}\,0 \quad V^2\cos^2\theta = \frac{1}{2} g S_x^2 + V^2\sin\theta\cos\theta \; S_x$$

$$\theta = \quad S_{\cancel{x}} = \frac{-2v^2 \sin\theta\cos\theta}{g}$$

$$S_x = -\frac{v^2 \sin(2\theta)}{g}$$

$$30 = \frac{\cancel{-}(35)^2 \sin(2\theta)}{\cancel{-}9.81}$$

$$\theta = \left(\sin^{-1}\left(\frac{30 \times 9.8}{35^2}\right)\right) \div 2$$



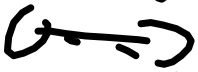
= 30x9.8=294

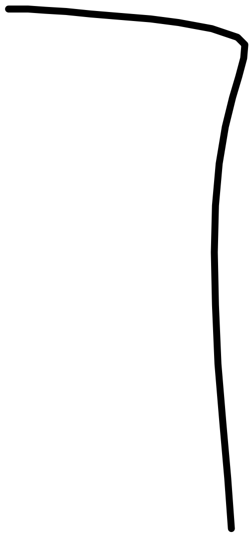
294/(35^2)=0.24

13.886/2=6.943

q49

V_0





$v_x = v_0$

$v_y = gt$

$$\boxed{\theta = \tan^{-1} \frac{gt}{v_0}}$$

Dynamics in 2-D (on test Oct 21)

Newton's 3 Laws:
First Law - Law of Inertia
- Objects stay at constant velocity (speed and direction) unless an unbalanced force is applied.
  unbalanced force: vector sum of all forces does not add to zero - vector diagram
  ie. the components do not add to zero.

  watch out! the net force is not a force, it is the sum of all forces. Don't put the net force on a free body diagram.

  Newton's Second Law: Law of acceleration.

$F_{net}=ma = \Delta p/\Delta t$

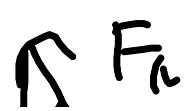
the acceleration of an object is proportional to the vector sum of all forces and inversely proportional to the mass of the object.

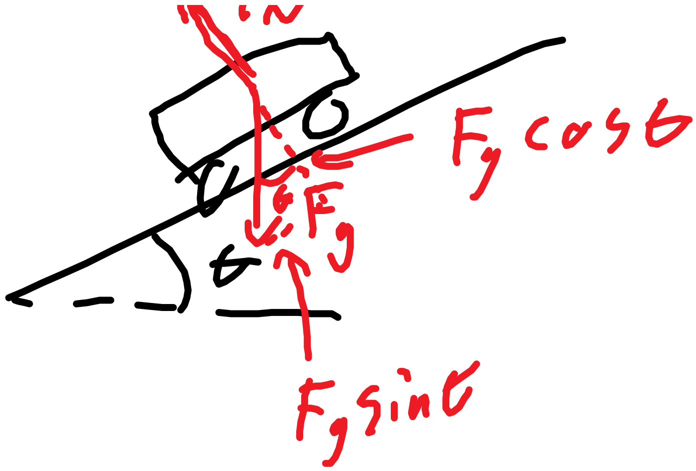
Newton's Third Law: Action-reaction Law.

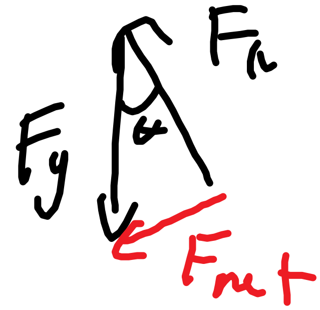
For every force object A acts on object B, B responds with an equal and opposite force on object A.

Look at a cart/block rolling/sliding down a slope. Draw a free body diagram of the forces on the cart, derive the acceleration of the cart if

a) frictionless
b) coefficient of kinetic friction is μ_k.
c) block is sliding up the slope with kinetic friciton, μ_k then stops with static friction μ_s.
d) solve a,b and c for 1.0kg object on a slope with 27.0º to the horizontal and μ_k= 0.15 and solve for minimum μ_s to have the object stop.







Free body

Vector Add.

$$F_{net} = F_g \sin\theta$$

$$\cancel{m}a = \cancel{m}g\sin\theta$$

$$\boxed{a = g\sin\theta}$$

frictionless slope

Block 2-4

Projectiles HW, any questions?

θ

V_x

$S_x = 30m$

$V_x = V\cos\theta$

$S_x = V_x t$

$S_y = \frac{1}{2} g t^2 + u_y t$

$t = \frac{S_x}{V_x}$

$S_y = \frac{1}{2} g \left(\frac{S_x}{V_x}\right)^2 + u_y \frac{S_x}{V_x}$

$S_y V_x^2 = \frac{1}{2} g S_x^2 + u_y S_x V_x$

$0 = \frac{1}{2} g (30)^2 + 35\sin\theta (30) 35\cos\theta$

$0 = 9.8(30^2) + 35^2 (30) 2\sin\theta\cos\theta$

$0 = g S_x^2 + V^2 \cancel{S_x} 2\sin\theta\cos\theta$

$0 = g s_x + v$

$S_x = -\frac{v^2 \sin 2\theta}{g}$ $\sin 2\theta$

$\frac{30(9.8)}{35^2} = \sin 2\theta$ $\boxed{\theta = 6.9^\circ}$

Dynamics in 2-D (chapter 4)

Write out Newton's 3 laws from memory

First Law: Law of Inertia
Objects remain at the same speed and direction unless acted upon by an external unbalanced force.
unbalanced: vector sum is not zero, vector diagram does not meet, the components do not add to zero.

F_{net}=vector sum of all forces on an object

Newton's Second Law: Law of Acceleration.

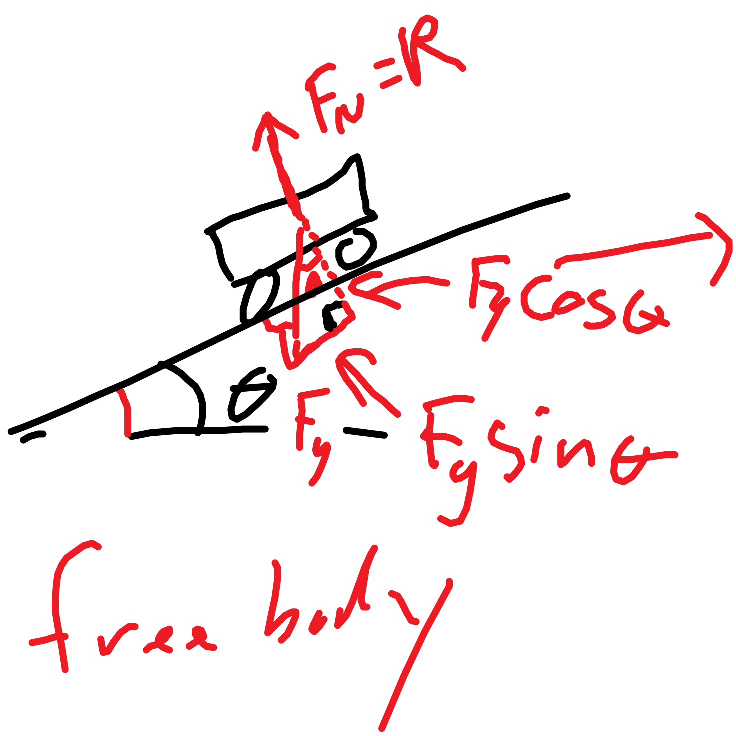
$F_{net} = ma = \Delta p/\Delta t$

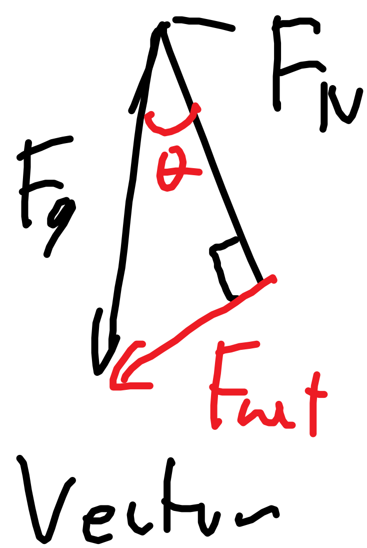
an object's acceleration is proportional to the vector sum of all force and inversely proportional to the mass.

slope of a p-t graph is F_{net}

Newton's Third Law: Action-reaction Law.

For every force object A acts on object B, object B responds with an equal and opposite force on object A.





$F_{net} = F_g \sin\theta$

$\cancel{m}a = \cancel{m}g \sin\theta$

$\boxed{a = g\sin\theta}$ frictionless slope

using a word processing document for the report, then save as pdf and e-mail to aklaassen@vsb.bc.ca
name and block in subject line
1 video per group - no data
1 lab report per person with data and 3 graphs
x-t, y-t and vy-t graphs

Projectile Lab

Name

Partners name:

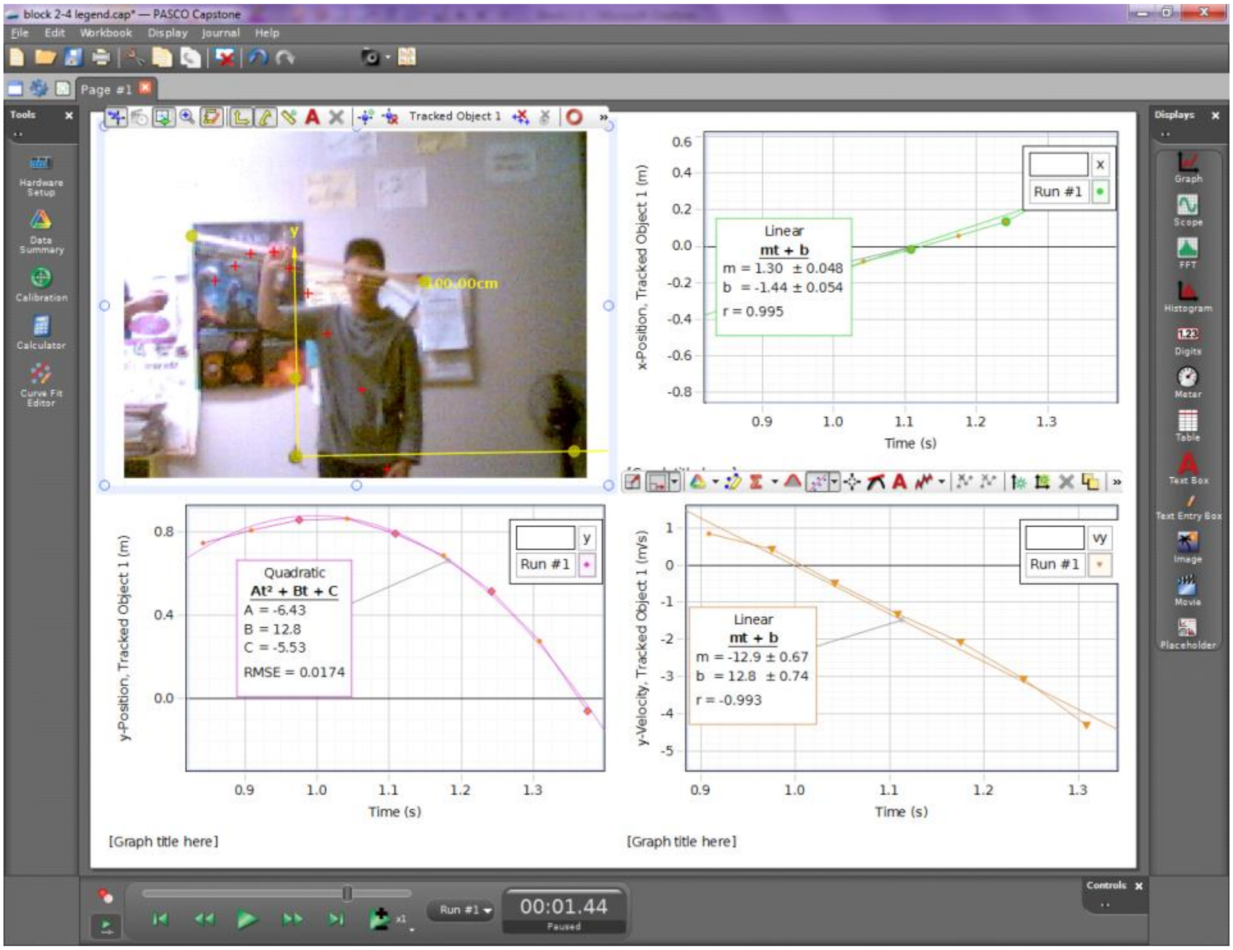
block

Purpose

Hypothesis

Procedure

Observations:



Analysis: d

Equation of each graph

$d_x = 1.30\text{m/s}\ t - 1.44\text{m}$

$d_y = 6.43\text{m/s}^2\ t^2 + 12.8\ \text{m/s}\ t - 5.5\text{m}$

$v_y = -12.9\text{m/s}^2\ t - 12.8\ \text{m/s}$

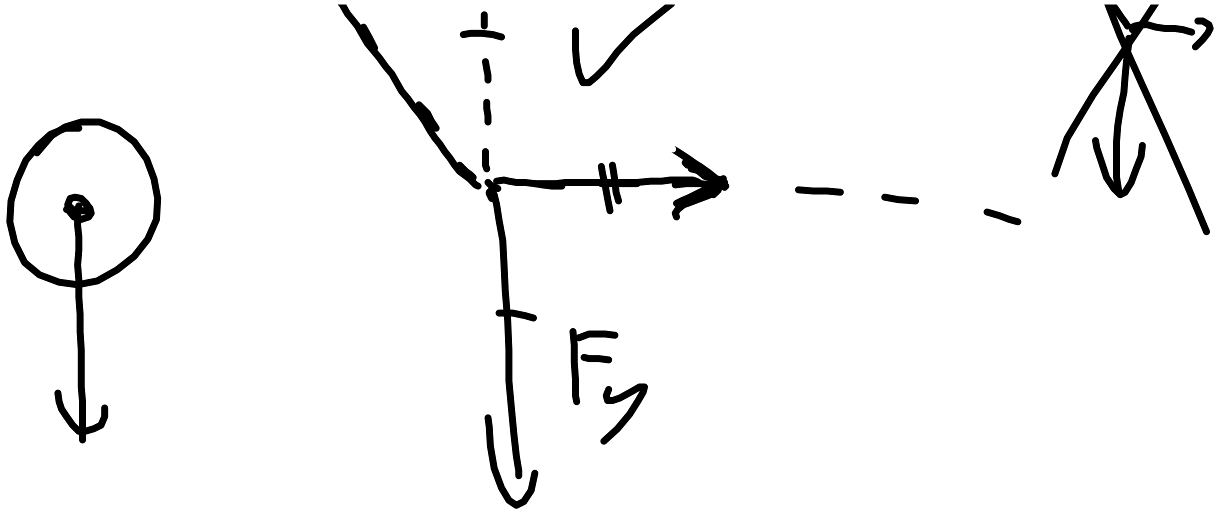
%error = |exp-theo|/theo x 100%

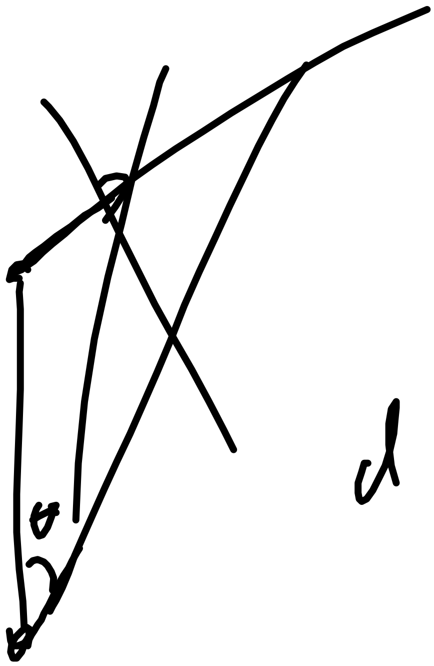
Eg. $|-12.9\text{m/s}^2 - -9.80\text{m/s}^2|/9.80 \times 100\%$

=

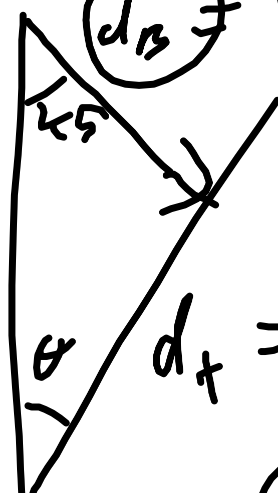
Conclusion- does each graph match hypothesis?

Sources of uncertainty- estimate of the quantity – evidence in the data





d



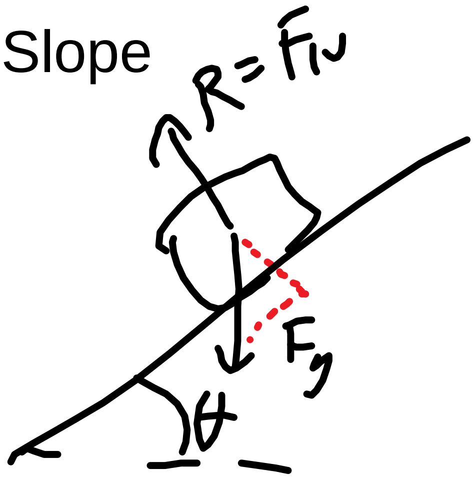
$d_B = V_B t$

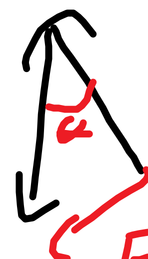
$d_t = V_t t$

$$\frac{\sin\theta}{V_B \cancel{t}} = \frac{\sin 45}{V_t \cancel{t}}$$

Monday, October 19, 2015
12:20 PM

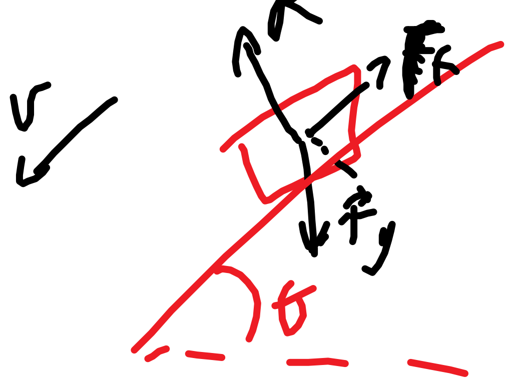
Block on a Slope

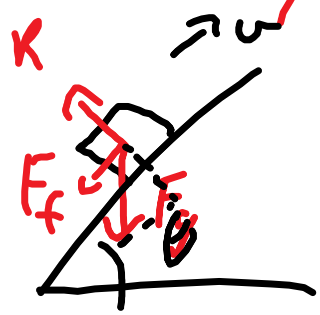




$\cancel{m}a = \cancel{m}g \sin\theta$

$a = g \sin\theta$



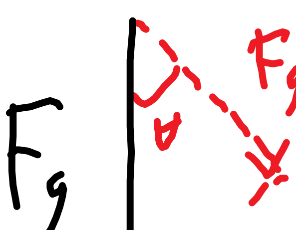


v down

v up

Component: Parall to slope, $\parallel$

or Perpendicular to slope $\perp$



$F_{g\perp} = F_g \cos\theta$

↓ $F_{g\parallel} = F_g \sin\theta$

for a block on a slope,

$$F_N = R = F_{g\perp} = F_g \cos\theta$$ (Not for Banking)

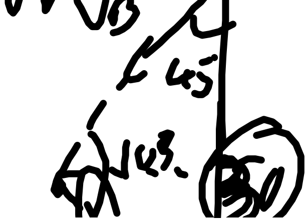
$$F_{net} = F_{g\parallel} \pm F_f \qquad F_f = \mu R$$

↑ depends on v

$$ma = mg\sin\theta \pm \mu mg\cos\theta$$

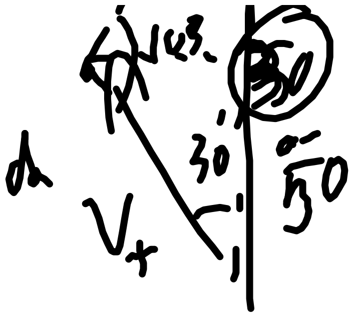
$$\boxed{a = g\sin\theta \pm \mu g\cos\theta}$$

12 m



$V_B \sin 45^\circ = V_T \sin 30^\circ$

$\underline{20\,km/s \sin 45} = \boxed{V_T = 28.3\,km/s}$



$$\frac{20\,km/s \sin 45}{\sin 30} = \boxed{V_t = 28.3\,km/h}$$

$$\frac{\sin 105}{50..30} = \frac{\sin 45}{d_r} = \frac{\sin 30}{d_1}$$

$$t = \frac{d_1}{20}$$

3

$V_y = 0$
$v = V_x = 5\,m/s$

$$V_f = \frac{d_r}{t} = \boxed{28.3\,km/h}$$

$$d_x = v_x t = 5 \times 1.0\,s \times 2 = 10\,m$$
$$\text{or } 5 \times 2.0\,s \times 2 = 20\,m$$

4.

$V_{yi} = V \sin\theta$
$V_x = V \cos\theta$

$$d_y = \frac{1}{2} g t^2 + V_{yi} t$$

$$-120 = -4.9 t^2 + 80 \sin 37^\circ\, t$$
$$\text{or } -160$$

$$t = 12\,s \qquad d_x = V \cos\theta\, t$$

(4.5

= 80 cos 37 12

or 14

79.0 m or 76.0 m

Look at a cart/block rolling/sliding down a slope. Draw a free body diagram of the forces on the cart, derive the acceleration of the cart if

a) frictionless = $g\sin\theta$

b) coefficient of kinetic friction is μ_k.

c) block is sliding up the slope with kinetic friciton, μ_k then stops with static friction μ_s.

d) solve a,b and c for 1.0kg object on a slope with 27.0º to the horizontal and μ_k= 0.15 and solve for minimum μ_s to have the object stop.

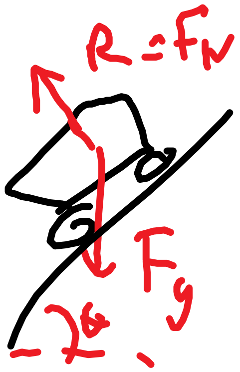
frictionless= 9.81m/s^2 sin 27 = 4.45m/s^2

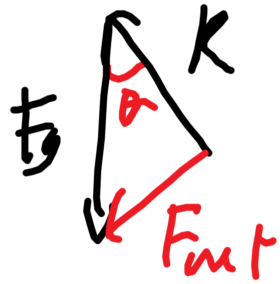
friction $g\sin\theta + \mu g\cos\theta$
= 9.81(sin27 + 0.15 cos27)
= 6.22 m/s^2

$g\sin\theta = \mu g\cos\theta$
$\mu=\tan\theta$ = tan 27º = 0.51

Look at a cart/block rolling/sliding down a slope. Draw a free body diagram of the forces on the cart, derive the acceleration of the cart if

a) frictionless



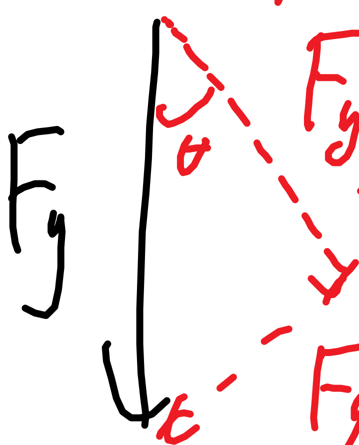


$F_{net} = F_g \sin\theta$

$ma = mg\sin\theta$

$\boxed{a = g\sin\theta}$

Perpendicular



$F_{g\perp} = F_g \cos\theta$

$F_{g\parallel} = F_g \sin\theta$

Parallel

b) coefficient of kinetic friction is μ_k.

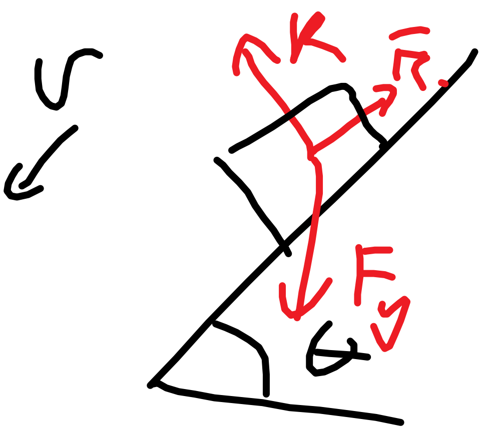
$F_f = \mu R$

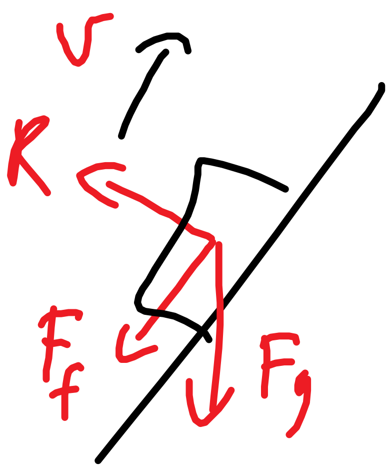
$F_f = \mu\, mg\cos\theta$ (Not valid Braking

-(circular)

c) block is sliding up the slope with kinetic friciton, μ_k then stops with static friction μ_s.

d) solve a,b and c for 1.0kg object on a slope with 27.0º to the horizontal and μ_k= 0.15 and solve for minimum μ_s to have the object stop.





$F_{net} = \Sigma F$ $\quad$ $F_\perp$ cancel

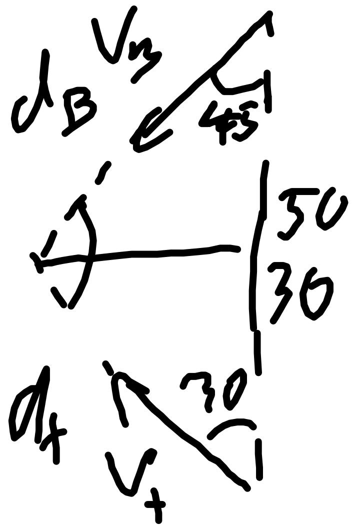
$F_{g\perp} = |K|$

$F_{net} = F_{g\parallel} \mp F_f$

$ma = mg\sin\theta \mp \mu mg\cos\theta$

$$\cancel{m}a = \cancel{m}g\sin\theta \pm \mu \cancel{m}g\cos\theta$$

$$\boxed{a = g\sin\theta \pm \mu g\cos\theta}$$



$$V_B \sin 45^\circ = V_t \sin 30^\circ$$

$$V_t = 28.3\,\text{km/h}$$

$$\frac{\sin 30}{d_B} = \frac{\sin 45}{d_t} = \frac{\sin 105}{50 \text{ or } 30}$$

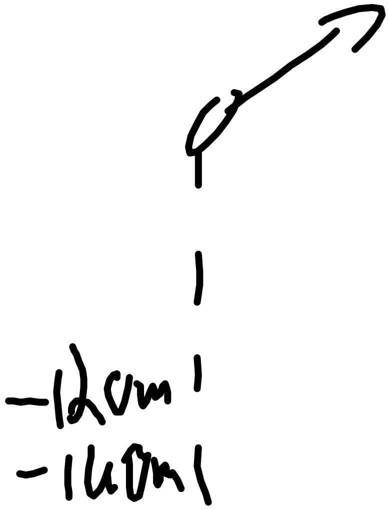
$$t = \frac{d_B}{20\,\text{km/h}} \qquad \frac{d_t}{t} = V_t$$

$V_y = 0$

$V_x = V = 5\,\text{m/s}$

$d_x = V_x t = 5\,m/s\,(1.0\,s \text{ or } 2.0\,s) \times 2$

$= 10\,m$ or $20\,m$



$d = \frac{1}{2} g t^2 + V_{yi} t$

-120 or $-140 = -4.9t^2 + 80\sin 37^\circ\, t$

$t = 12\,s$ or $14\,s$

$d_x = V\cos\theta\, t = 80\cos 37\ (12 \text{ or } 14)$

$760\,m$ or $890\,m$

$d_y = S_y$

$d_x = S_x$

$S_y = \frac{1}{2} g t^2 + V\sin\theta\, t$

$S_x = V\cos\theta\, t$

$t = \frac{S_x}{V\cos\theta}$

$$t = \frac{\;}{V\cos\theta}$$

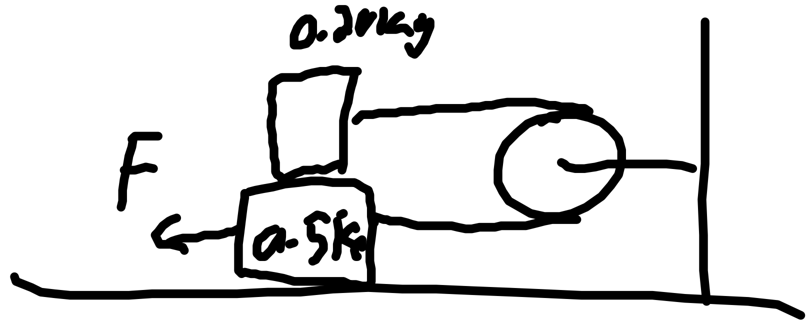
$$S_y V^2 \cos^2\theta = \frac{1}{2} g S_x^2 + V^2 (\sin\theta\cos\theta) S_x$$

Pulley Problems

A 0.50kg mass and a 0.20 kg mass are suspended over a massless and frictionless pulley. Determine the acceleration and tension in the string if

a) system is vertical and hanging freely
b) 0.50 kg mass is on a frictionless slope, 70.0º to the horizontal.

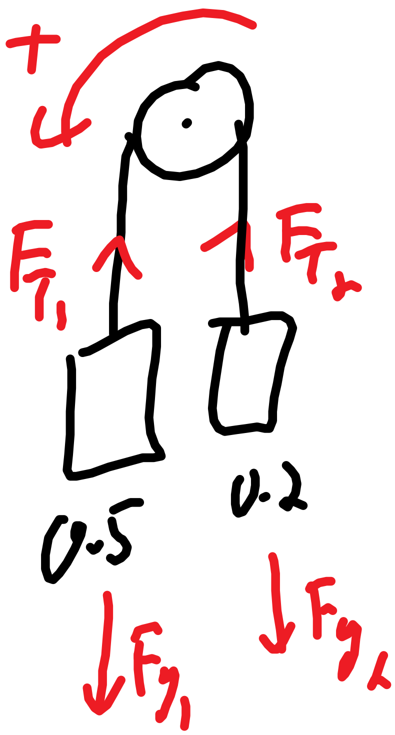
c) same as b but with μ=0.40
d) horizontal system with the 0.20kg mass on the 0.50 kg mass, μ=0.40 between all surfaces.



if F=15.0N, det
ermine a, F_t.
p91-92 Questions 1,3,4,11,17,21

p94-96 Problems 27,29,31,35,39,47,49,51

a)



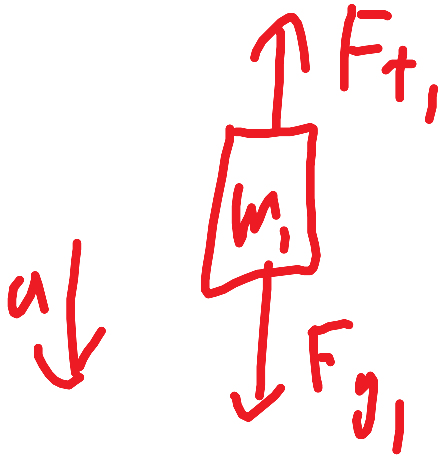
$$\sum F = F_{g_1} - F_{g_2}$$

$$m_t a = m_1 g - m_2 g$$

$$\boxed{a = \frac{m_1 - m_2}{m_1 + m_2} g}$$

$$a = \frac{0.5 - 0.2}{0.5 + 0.2} 9.81$$

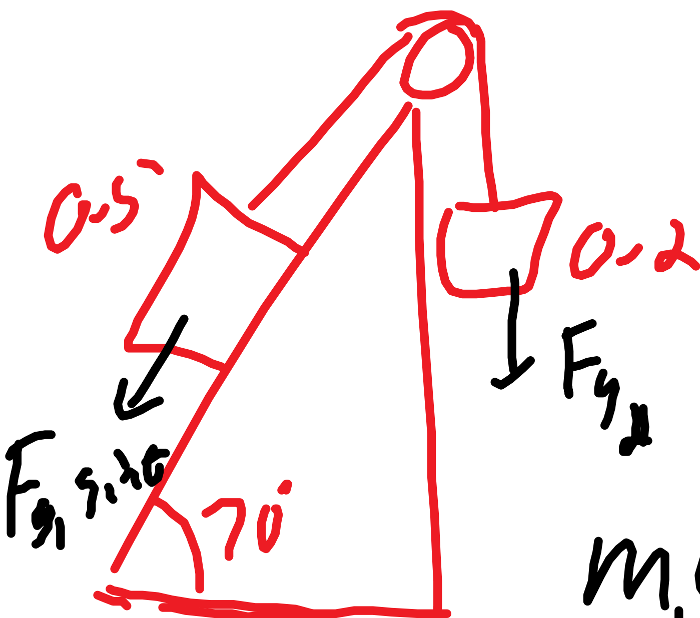
$$\boxed{a = 4.2 \ m/s^2} \quad 4.20428$$



$$\boxed{m_1 a = F_{g_1} - F_{T_1}}$$

$$0.5(4.20) = 0.5(9.81) - F_{T_1}$$

$$\boxed{F_{t_1} = 2.8\,N}$$



$$\Sigma F = F_{g_1}\sin\theta - F_{g_2}$$

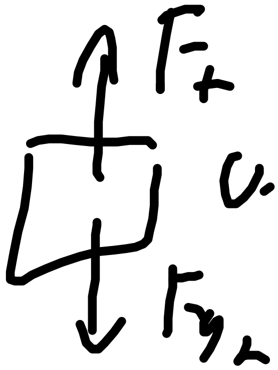
$$m_T a = m_1 g \underline{\underline{\sin\theta}} - m_2 g$$

$$a = \frac{0.5(9.8)\sin 70 - 0.2(9.8)}{\;}$$

$$a = \frac{0.5(\ldots)}{0.2 + 0.5}$$

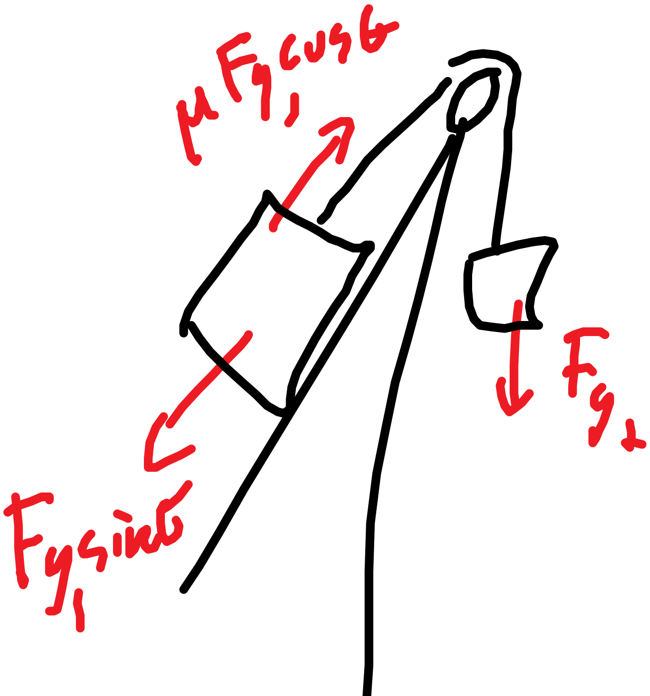
$$\boxed{a = 3.8\ m/s^2}$$

* $a \uparrow$



$0.2(3.8) = F_t - 0.2 \times 9.8$

$$\boxed{F_t = 2.7\ N}$$



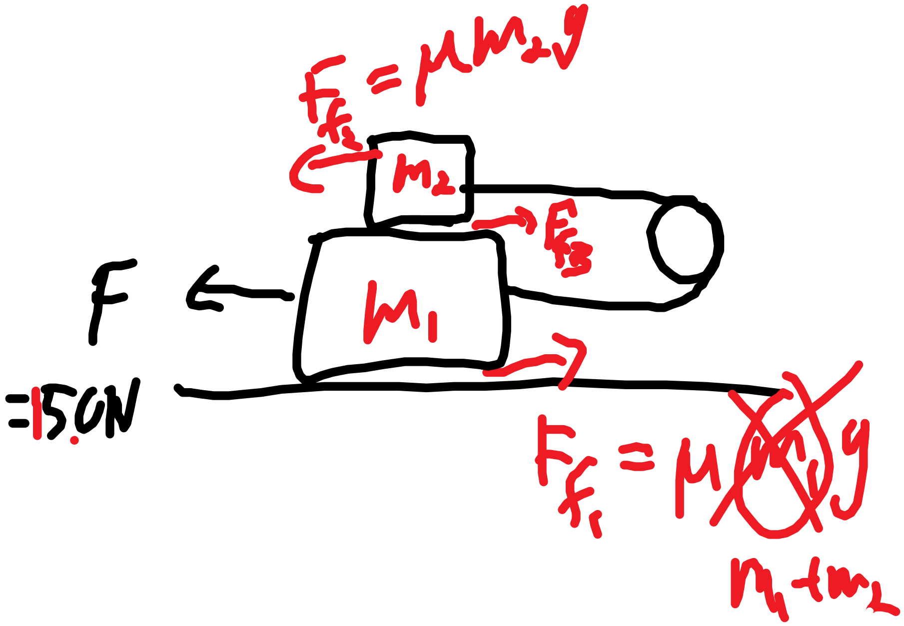
$\Sigma F = F_{g_1}\sin\theta - \mu F_{g_1}\cos\theta - F_{g_2}$

$$m_T a = [m_1(\sin\theta - \mu\cos\theta) - m_2]g$$

$$a = \frac{[0.5(\sin 20 - 0.4 \cos 20) - 0.2]\, 9.81}{0.2 + 0.5}$$

$$\boxed{a = 2.8\ m/s^2}$$

$$\boxed{F_t = 2.5\ N}$$



p91-92 Questions 1,3,4,11,17,21
p94-96 Problems 27,29,31,35,39,47,49,51

$$F_{net} = \Sigma F = 50N - (0.2 + 0.5)\mu\, 9.8 \quad (\mu = 0.40)$$

$$2(0.2)(9.8)0.4$$

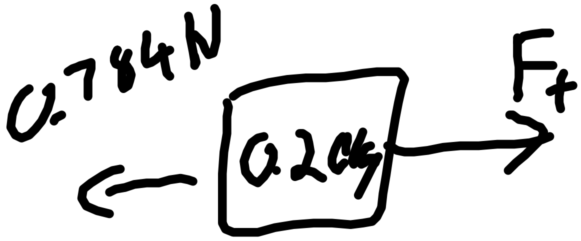
both $m_1 + m_2$

$$50N - 2.7448 - 1.568$$

$$F_{net} = 45.6852N = 0.7kg\,a$$

$$\boxed{a = 65.3 m/s^2}$$

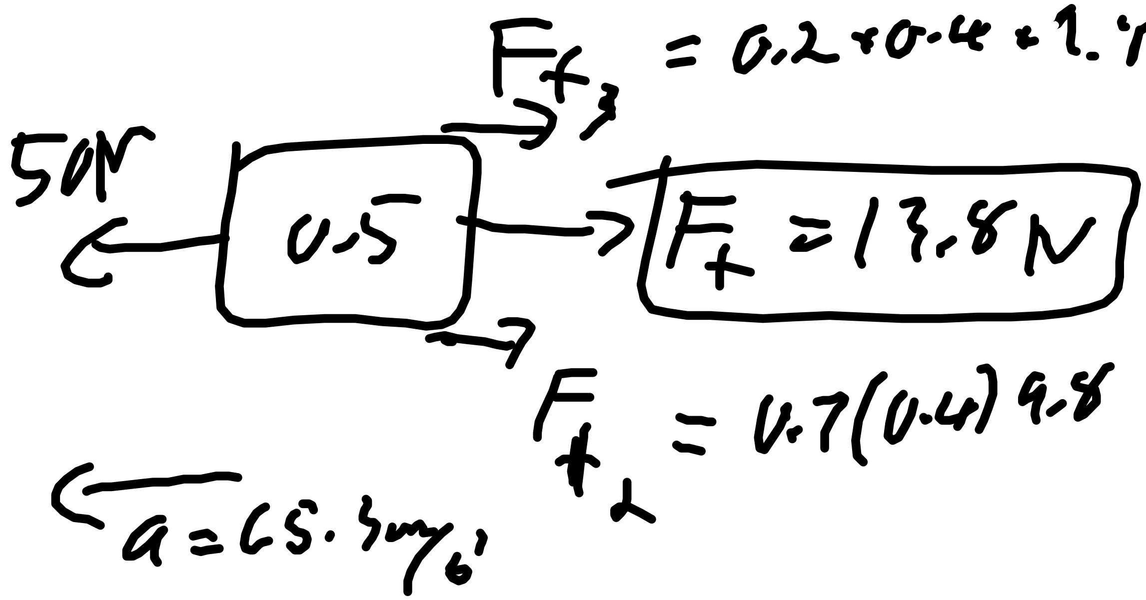
$\rightarrow a = 65.3 m/s^2$



$$F_{net} = ma = F_t - F_f$$

$$F_t = 0.2(65.3) + 0.784$$

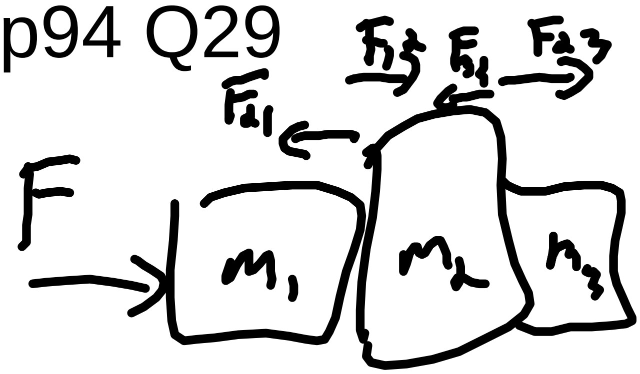
$$\boxed{F_t = 13.8N}$$



$$0.5 \times 65.3 = 50 - F_{f3} - F_{f2} - F_L$$

$$32.65 - 50 + 0.78 + 2.744 = \boxed{-13.8N}$$

p94 Q29



$$a = \frac{F_{net}}{m}$$

$$a = \frac{F}{m_1 + m_2 + m_3}$$

b) m_1 $\quad F_{net} = m_1 a = \dfrac{m_1 F}{m_1 + m_2 + m_3}$

c) $F_{23} = \dfrac{m_3 F}{m_1 + m_2 + m_3}$ only force on m_3

$$F - F_{21} = F_{net} = m_1 a$$

$$F_{21} = F - \frac{m_1 F}{m_1 + m_2 + m_3}$$

$$F_{21} = \frac{F(m_1 + m_2 + m_3) - m_1 F}{m_1 + m_2 + m_3}$$

$$F_{21} = \frac{(m_2 + m_3) F}{m_1 + m_2 + m_3}$$

$m_1 + m_2 + m_3$

Wednesday, October 28, 2015
12:24 PM

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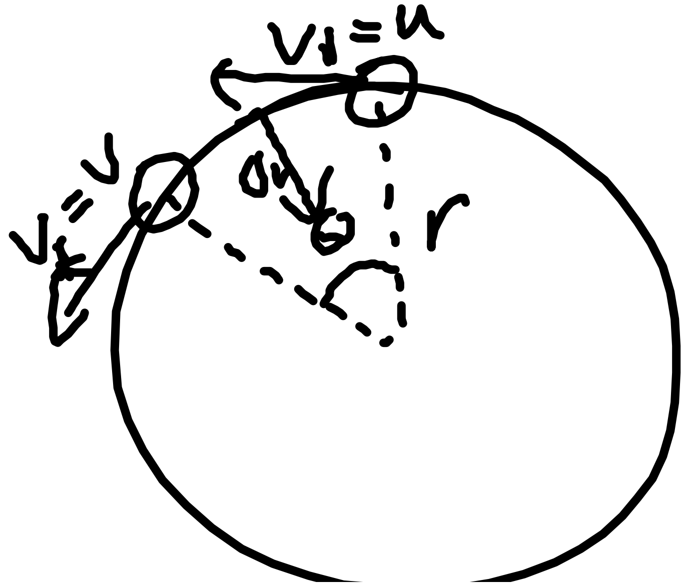
Circular Motion (ch5)
motion in a circular path
uniform circular motion- constant speed
(note: velocity is constantly changing)
non-uniform - speed changes but still circular

if v is changing then there must be acceleration.
a=Δv/Δt
for uniform acceleration a=(v-u)/t

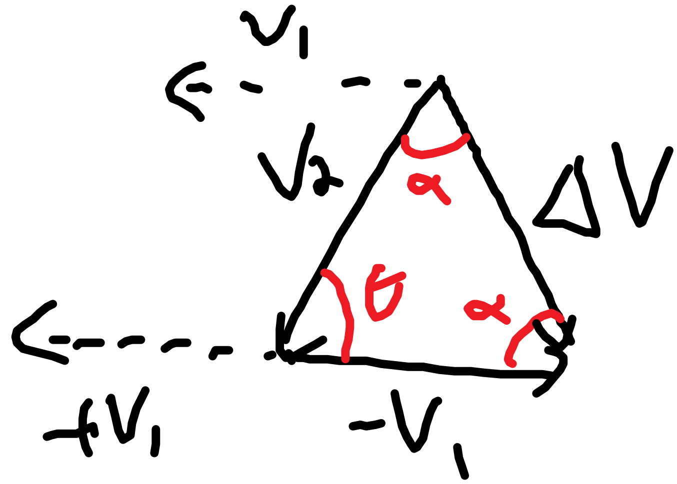
but the magnitude of v and u are the same, what's the deal?
vector subtraction



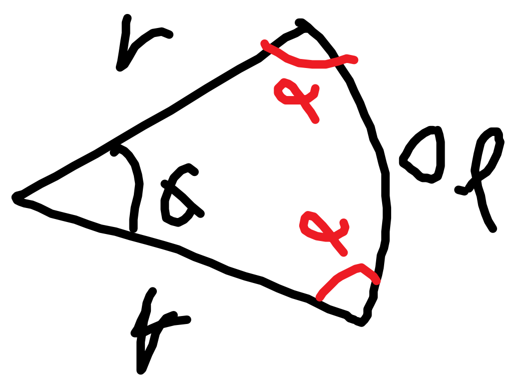
for ~~uniform~~ circular motion

$V \perp r$

$\Delta V = V_2 \ominus V_1$



Simila Δs



$$\frac{\Delta V}{V} = \frac{\Delta \ell}{r}$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = \frac{\Delta \ell}{r} V$$

$$a = \frac{\Delta \ell}{\Delta t} \frac{V}{r}$$

$$v = \frac{\Delta \ell}{\quad}$$

$$\boxed{a = \frac{v^2}{r}} \leftarrow \text{for uniform circular motion} \qquad v = \frac{\Delta \ell}{\Delta t}$$

the acceleration is towards the centre of the circular path.
even though the speed is constant, there is acceleration.

Trust me
for non-uniform circular motion, the radial component of the acceleration, is the centripetal acceleration, $a_c=v^2/r$ using the speed at that instant.

eg. I will swing a bucket of water over my head.

a) If the radius of the circular path is 1.0m what is the minimum speed that I should swing the bucket so I don't dump water

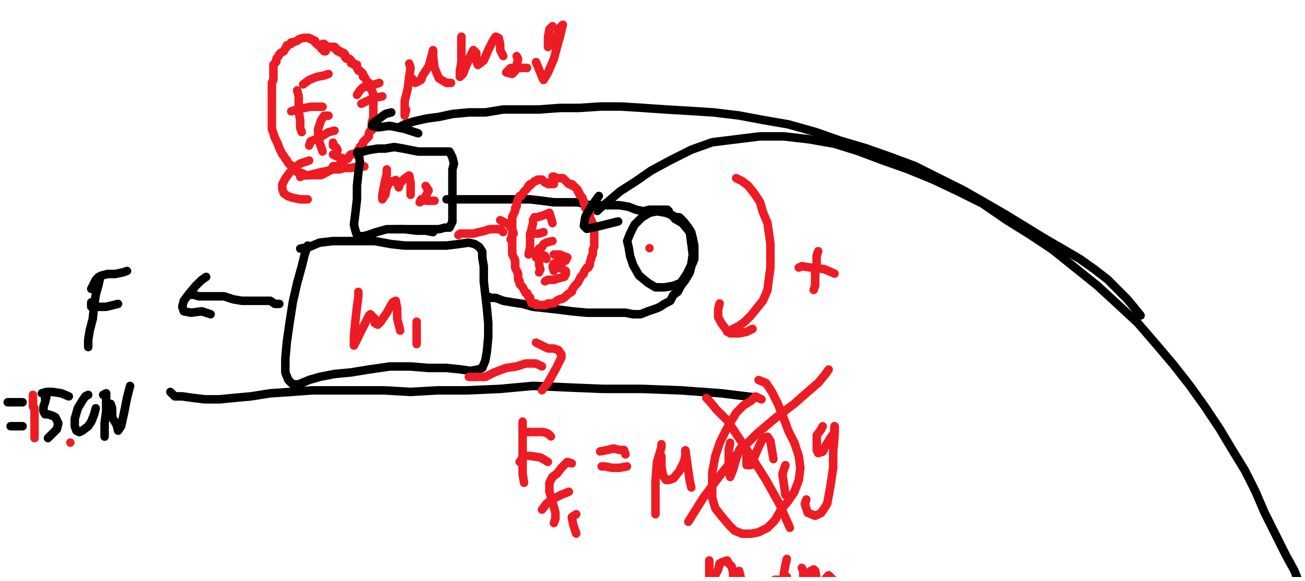
on my head?

b) Why doesn't the water spill out?

c) What is the period of revolution to give the minimum speed at the top of the path?

d) What is the tension in my arm if there is 2.0kg of water in the bucket at i)top ii) halfway down iii) bottom (assume speed is constantly at the minimum speed)

e) If the speed at the top is the minimum speed and the bucket accelerates due to gravity (non-uniform) what is the speed and tension at the bottom of the circular path.

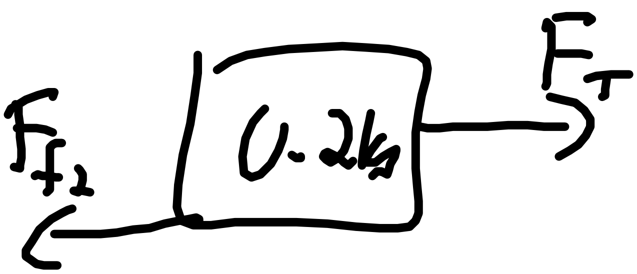


m_1+m_2

$$15.0N - 0.4(0.7)9.8 - (0.2(0.4)9.8)\times 2$$

15.0N-2.744N-2x0.784= 10.688N
a=Fnet/m = 10.688N/0.7kg
10.688/0.7=15.2686

$\boxed{15.3m/s^2}$

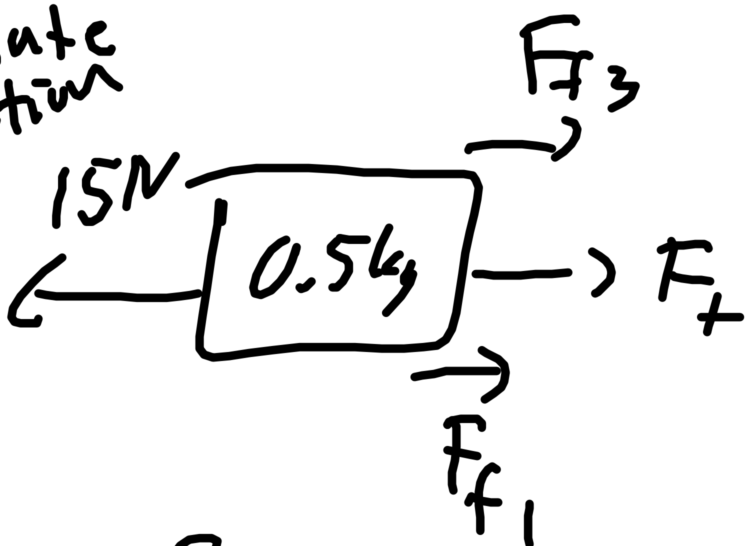


$$F_{net} = ma = F_T - F_{f_2}$$

$$F_t = 0.2\,\text{kg}\left(15.3\,\text{m/s}^2\right) + 0.2(9.8)0.4$$

$$= \boxed{3.84\,\text{N}}$$

alternate solution



$$\Sigma F = ma$$

$$15\,\text{N} - F_{f1} - F_{f3} - F_t = 0.5(15.3)$$

$$15 - (0.4 \times 0.7 \times 9.8) - (0.2 \times 0.4 \times 9.8) - F_t = 7.52$$

$$\boxed{F_t = 3.84\,\text{N}}$$

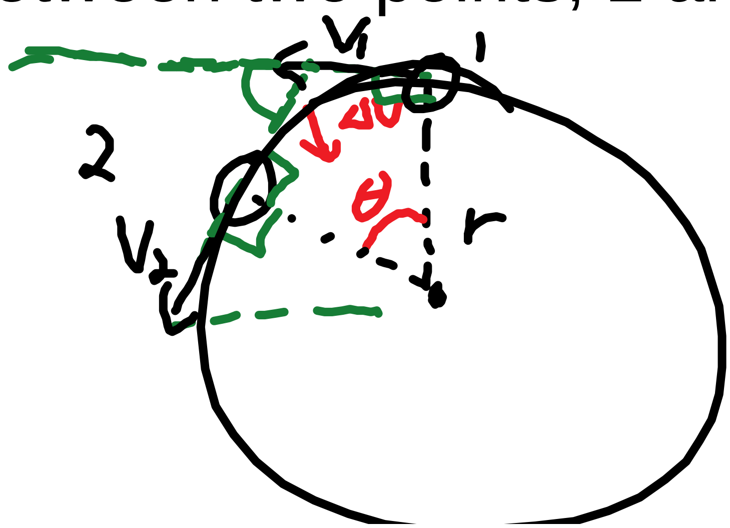
Circular Motion (chapter 5)

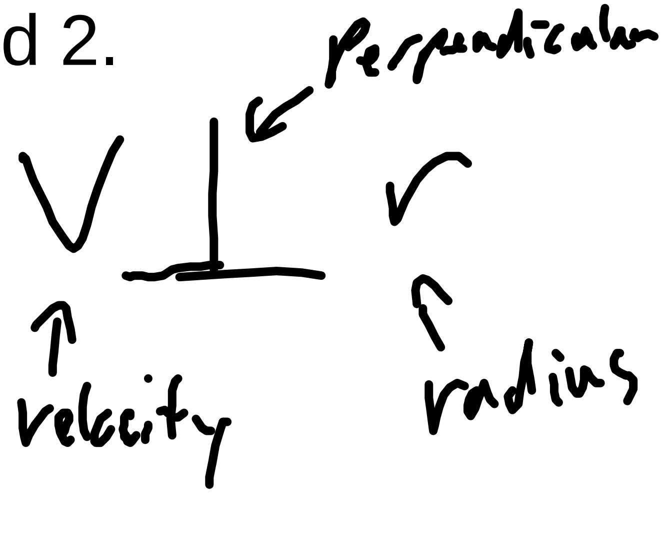
Motion in a circular path.
Uniform circular motion: motion with a constant speed. (the velocity is constantly changing as the direction changes - so there is acceleration)

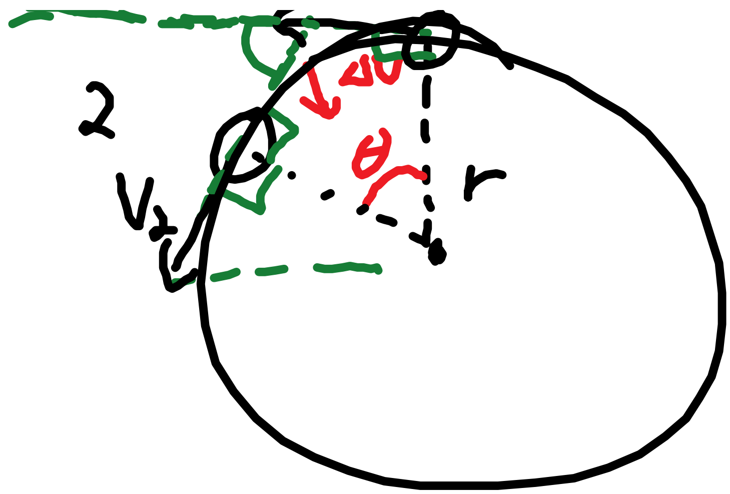
non-uniform circular motion - circular path but changing speed.

Bucket - I swing a bucket of water over my head. Why doesn't the water fall out?

look at an object moving in uniform circular motion - the velocity of the object changing between two points, 1 and 2.





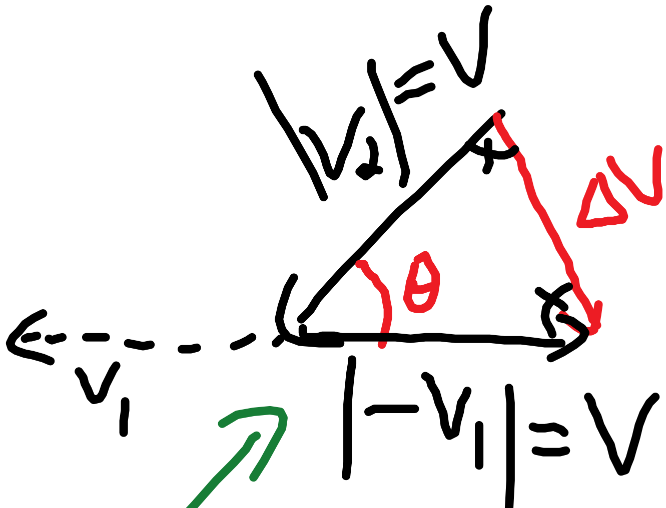


$V \perp r$

velocity, radius

$a = \dfrac{\Delta V}{\Delta t}$ for uniform a

$= \dfrac{V_2 - V_1}{t}$ Vector subtraction



$\Delta V \leftarrow$ towards the centre of the circle

Similar Δs

$\leftarrow$ arc length

$\dfrac{\Delta V}{\ } = \dfrac{\Delta l}{\ }$ $a = \dfrac{\Delta V}{\ }$

$$\frac{\Delta v}{v} = \frac{\Delta \ell}{r} \qquad a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = \frac{\Delta \ell \, v}{r}$$

$$a = \frac{\Delta \ell \, v}{\Delta t \, r} \qquad v = \frac{\Delta \ell}{\Delta t}$$

$$\boxed{a = \frac{v^2}{r}}$$

Uniform circular motion acceleration $= a = \frac{v^2}{r}$

v is speed, r is radius

of the circular path.
a is towards the centre.

calculus magic (trust me)
for non-uniform circular motion, the component of the acceleration towards the centre of the circular path $a_c = v^2/r$

a_c is the centripetal acceleration
v is the instantaneous speed
r is the radius of circular path

Friday, October 30, 2015
12:28 PM

eg. I will swing a bucket of water over my head.

a) If the radius of the circular path is 1.0m what is the minimum speed that I should swing the bucket so I don't dump water on my head?

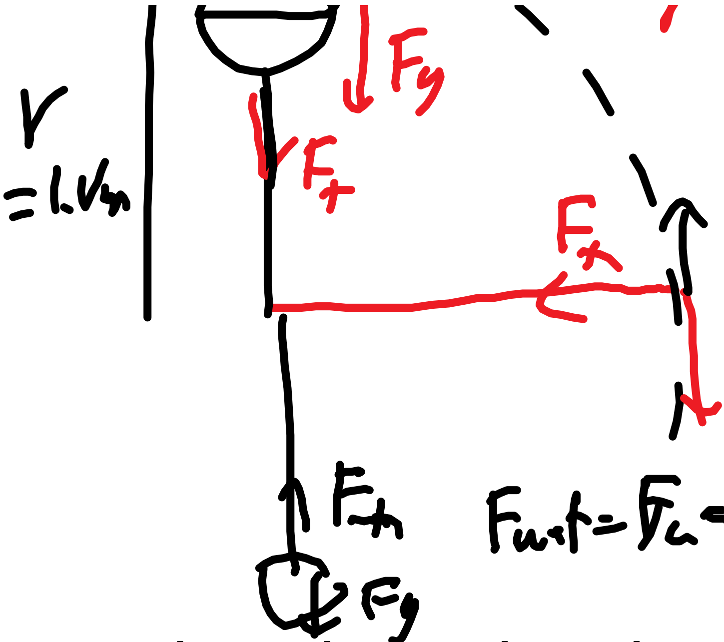
b) Why doesn't the water spill out?

c) What is the period of revolution to give the minimum speed at the top of the path?

d) What is the tension in my arm if there is 2.0kg of water in the bucket at i)top ii)halfway down iii) bottom (assume speed is constantly at the minimum speed)

e) If the speed at the top is the minimum speed and the bucket accelerates due to gravity (non-uniform) what is the speed and tension at the bottom of the circular path.

Top $F_{net} = F_T + F_g$



$F_{net} = F_c = F_t - F_g$

$F_t = F_c \ominus F_g$

$F_t \perp F_g$

$F_t = F_c$

centripetal acceleration a_c is the component of acceleration towards the centre of the circle

$F_{net} = ma$ so

centripetal force is the component of the net force towards the centre of the circular path

- it is not a force - it is the sum of all forces.

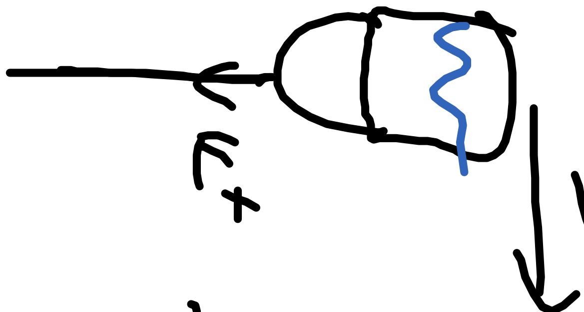
the water doesn't go out of the bucket because as it falls, it moves sideways and down

what is the rate at which the water falls?

at least $9.81 m/s^2 = v^2/r$ at minimum speed

$$v = \frac{\sqrt{9.81m \times 1.0m}}{s^2}$$
$= 3.1 m/s$

i) at the top if we are at minimum speed, the acceleration of the water is $9.81 m/s^2$ so no force is required - gravity is the centripetal force.
$F_t = 0$



$v = 3.1 m/s$
assume uniform speed

$$F_T = \frac{mv^2}{r} = \frac{2.0kg(3.1)^2}{1.0m}$$

$$= 19.6 N = \boxed{20.N}$$

iii) at the bottom, Fnet = Fc = Ft - Fg
Ft = Fc+Fg = 20N+20N=<u>40N</u>

c) $v = d/t = 2\pi r/T$
$a = v^2/r = (2\pi r/T)^2/r =$ $\frac{4\pi^2 r^2}{T^2} \times \frac{1}{r}$

$\boxed{a = 4\pi^2 r/T^2} = 4\pi^2 r f^2$

$a = v^2/r = (2\pi r/T)^2/r = \frac{4\pi^2 r}{T^2}$

$\boxed{a=4\pi^2 r/T^2} = 4\pi^2 r f^2$

uniform only

$$T = \sqrt{\frac{4\pi^2 r}{a}} = 2\pi\sqrt{\frac{r}{a}}$$

$$= 2\pi\sqrt{\frac{1.0\,m}{9.81\,m/s^2}} = \boxed{2.0\,s}$$

V at top $= 3.1\,m/s$

V at bottom?

$$\cancel{2}\cdot\frac{1}{\cancel{2}}\cancel{m}v_b^2 = 2\cancel{m}gh + \cancel{2}\cdot\frac{1}{\cancel{2}}\cancel{m}v_{top}^2$$

$$V_b = \sqrt{2(9.81)(2) + (3.1)^2}$$

$$V_b = 6.99\,m/s$$

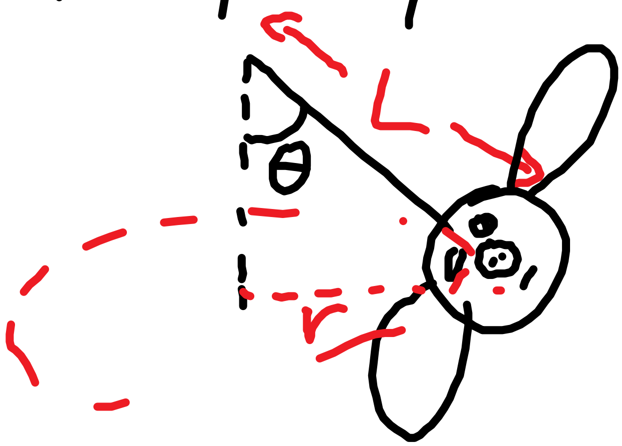
$$F_t = \frac{mv^2}{r} + F_g = \frac{2(6.99)^2}{\ } + 2(9.81)$$

$F_T = \frac{\ldots}{r} + F_g \ldots \frac{\ldots}{l} + 2(1.8)$

$= \boxed{117\,N}$

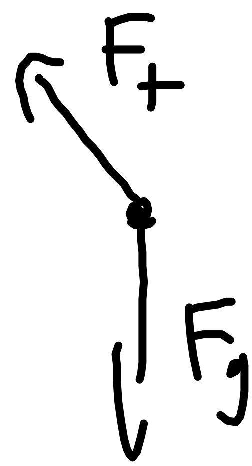
Flying Pig Lab

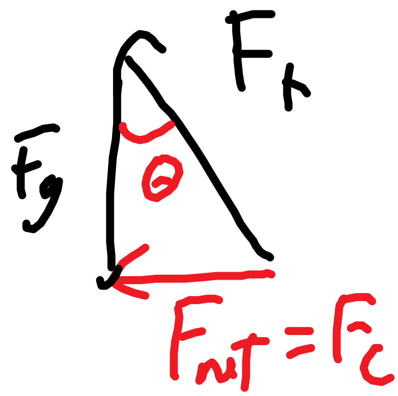
theory



L is distance from rotation point to centre of mass

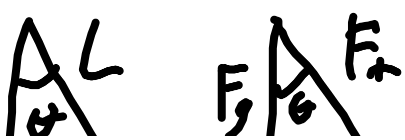
Apparatus



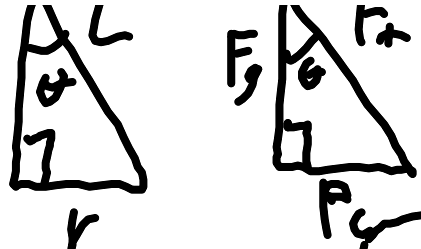


free body

Vector Add



banking



banking

$$\frac{r}{l} = \sin\theta \sim \boxed{\tan\theta = \frac{F_c}{F_g}}$$

for small θ $\frac{r}{L} = \frac{F_c}{F_g} =$

$$F_c = \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2}$$

$$\frac{\cancel{r}}{L} = \frac{\cancel{m}4\pi^2\cancel{r}}{\cancel{m}gT^2}$$

$$\boxed{T^2 = \frac{4\pi^2}{g}L}$$

Graph T^2 vs L

slope = $\frac{4\pi^2}{g}$

eg. I will swing a bucket of water over my head.

a) If the radius of the circular path is 1.0m what is the minimum speed that I should swing the bucket so I don't dump water on my head?

$F_{net} = F_c = F_t + F_g$

at minimum speed

$F_t = 0$

$F_c = F_g$

$\frac{\cancel{m}v^2}{r} = \cancel{m}g$

$v = \sqrt{rg}$

$= \sqrt{1.0m \cdot 9.81 m/s^2}$

$= \boxed{3.1 m/s}$

$= \boxed{3.1\ m/s}$

b) Why doesn't the water spill out?
The water is moving fast enough that it falls in a trajectory that follows the circular path. (like an orbit, circular path is caused by gravity and no other forces)

c) What is the period of revolution to give the minimum speed at the top of the path?

$v=d/t$ if speed is constant

$d=2\pi r \quad t=T$ period

$v=2\pi r/T$

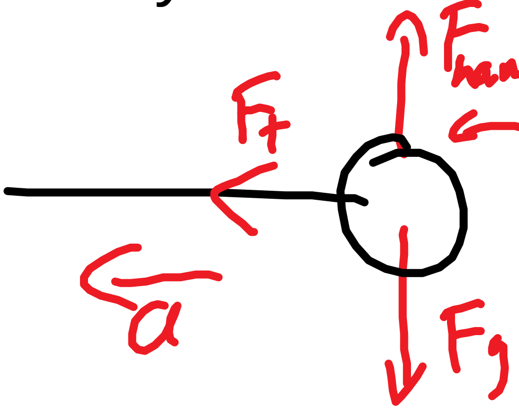
$a=v^2/r = (2\pi r/T)^2/r = 4\pi^2 r^{\cancel{2}} /T^2 \cancel{r}$

$\boxed{a=4\pi^2 r/T^2}$ $\quad T = 2\text{pi root}(r/a) = 2.0\text{s}$

d) What is the tension in my arm if there is 2.0kg of water in the bucket at i)top

$F_t = 0$ at minimum speed

ii)halfway down

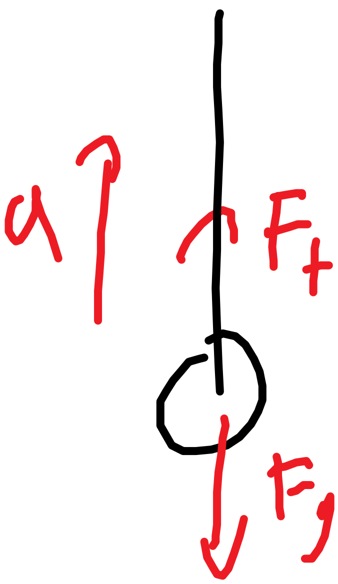


F_{hand} if v is constant then

$F_{net} = ma = F_T$

$= \underline{20N} = \dfrac{2kg(3.1 m/s)^2}{1.0m}$

iii) bottom (assume
speed is constantly at the minimum
speed)



$F_{net} = F_c = ma = F_T - F_g$

$F_T = ma + F_g$

$= 20N + 20N = \boxed{40N}$

a) If the speed at the top is the minimum speed and the bucket accelerates due to gravity (non-uniform) what is the speed and tension at the bottom of the circular path.

instantaneous v at bottom, v_b

$$\cancel{\tfrac{1}{2}}\cancel{m}v_b^2 = \cancel{m}2gh + \cancel{\tfrac{1}{2}}\cancel{m}v_{top}^2$$

$$V_b = \sqrt{2(9.81)(2) + (3.1)^2} \qquad \sqrt{2g2r + gr}$$

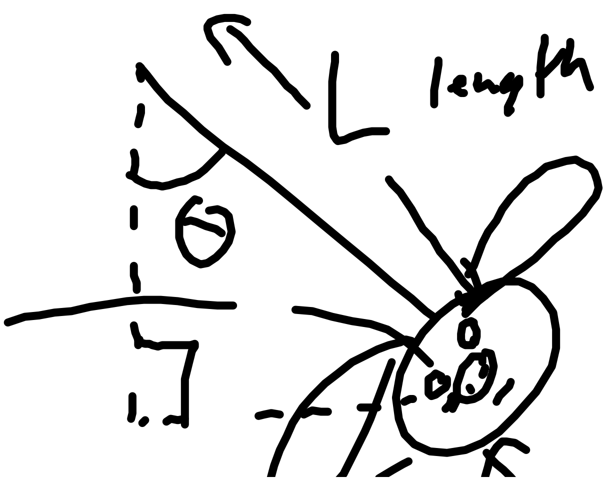
$$\sqrt{3gr}$$

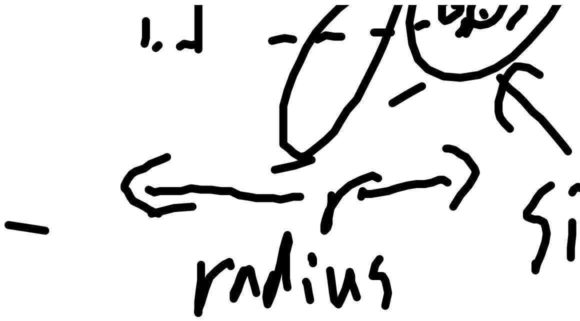
$$V_b = 7.0\,m/s \qquad \sqrt{5gr}$$

$$F_T = F_c + F_g = \frac{2(7)^2}{1} + 2(9.8)$$

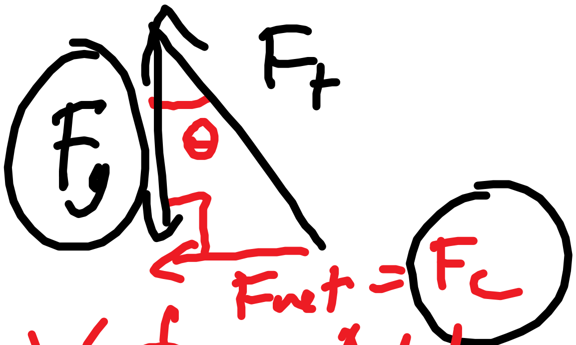
$$= \boxed{118\,N}$$

Flying Pig Lab





$\sin\theta = \frac{r}{L}$ free body



Vector Add

$$\boxed{\frac{F_c}{F_g} = \tan\theta} \approx \sin\theta = \frac{r}{L}$$

for small angles.

banking

$$\frac{F_c}{F_g} = \frac{\cancel{r}}{L} = \frac{\cancel{m}\,4\pi^2 \cancel{r}}{\cancel{m}\,g T^2}$$

$$\boxed{T^2 = \frac{4\pi^2}{g} L}$$

title
name, partners name, block
purpose: - specify variables
hypothesis - derivation from last class
$T^2 = 4\pi^2/g \; L$
procedure- write a procedure

observations:
table with units and uncertainties

time for 3 swings s +/-	$T^2(s^2)$ +/-	L(m) +/-
		0.50
		1.50

independent: L on x axis
graph T^2 vs L
% deviation of slope (units) to $4\pi^2/g$
write out your equation of the graph

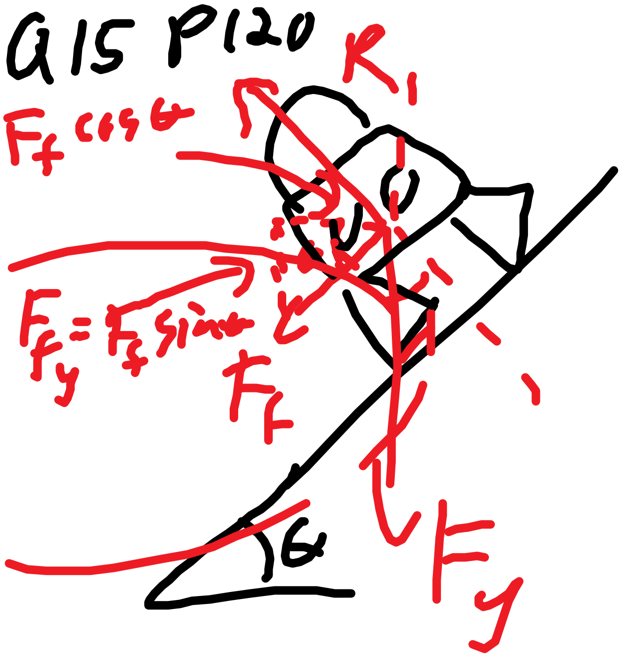
max/min lines

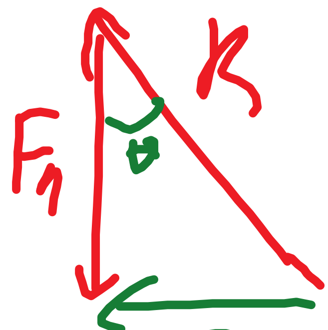
$T^2 = (4.0\ s^2/m \pm 0.1 s^2/m)\ L + 0.50 s^2 \pm 0.06 s^2$

conclusion - does the data support your hypothesis and to what precision.

Sources of error - quantify and compare to the deviation and give evidence.

Q15 P120





$F_{net} = F_c$

$\tan\theta = \frac{F_c}{F_g} = \boxed{\frac{v^2}{rg}}$

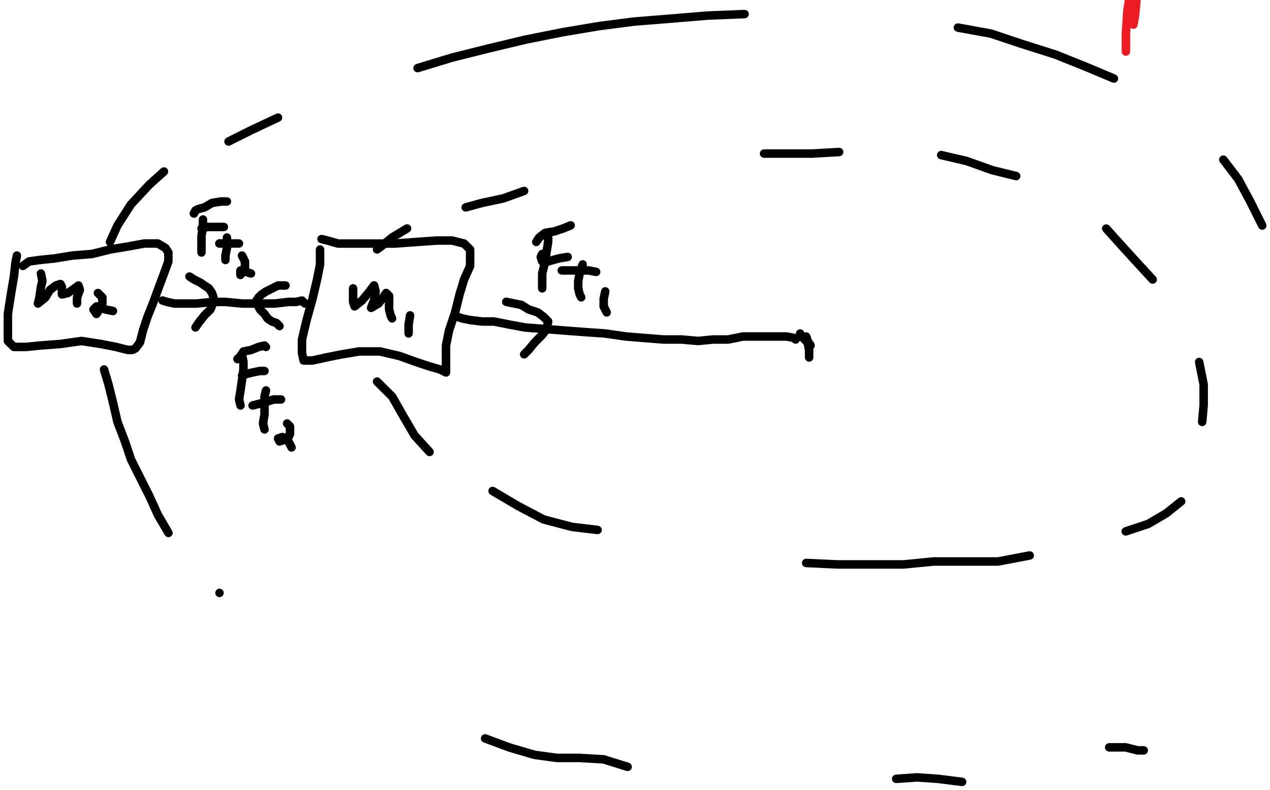
$\theta = \tan^{-1} \frac{(60/3.6\ m/s)^2}{60m\ 9.81\ m/s^2} = 25.26°$

$F_f = \mu F_N$

mg

y: $\mu K \sin\theta + F_y = K\cos\theta$

x: $F_c = \mu K\cos\theta + K\sin\theta = \frac{mv^2}{r}$



Gravity
definition
History - Aristotle - water and earth go down, fire and air go up. More earth (more mass) go down faster.
Galileo - found stuff falls at the same rate regardless of mass.
Kepler - Laws of planetary motions.

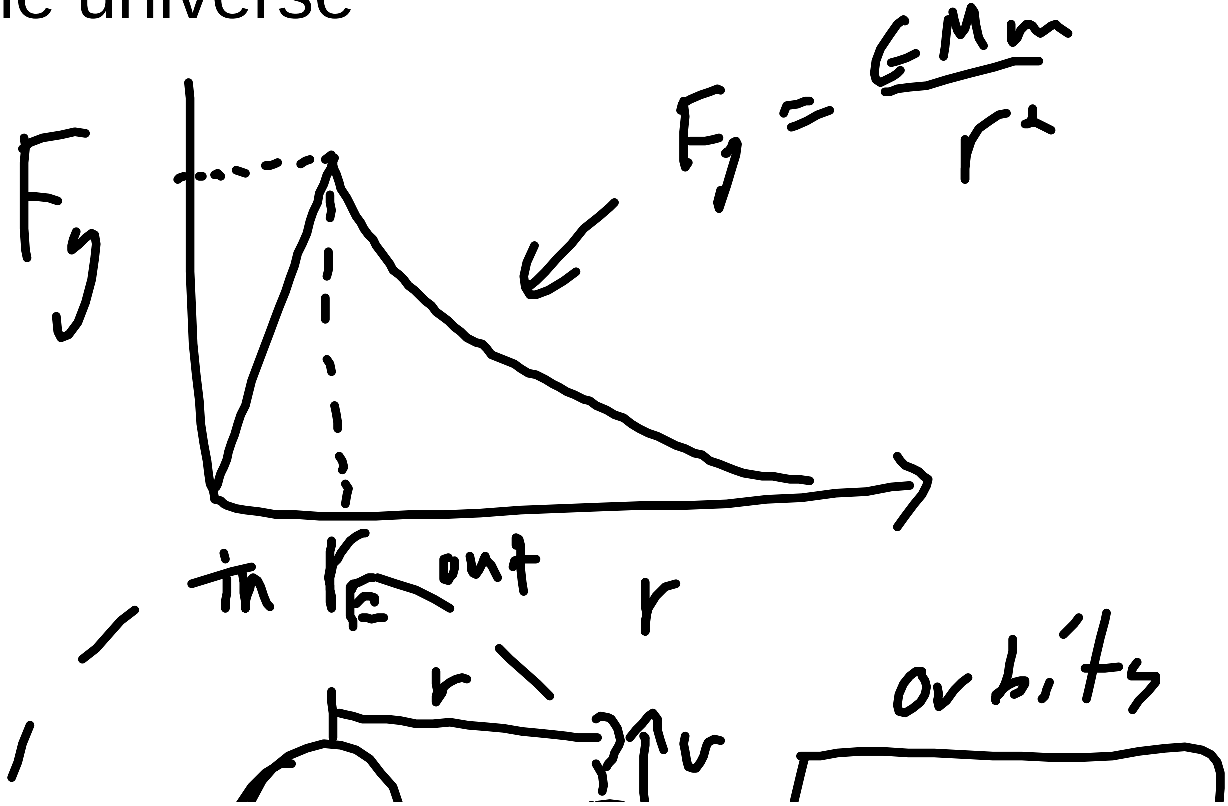
- planets move in ellipses with Sun at foci
- line from sun to planet sweeps out equal areas in equal time
- ratio - Periods of planets to their radii corresponds to $T^2/r^3 = k$ a constant
  Newton - everything is attracted to everything else. More mass = more attraction. More distance = less attraction.
  Einstein - gravity is curvature of space-time
  modern physics - graviton?

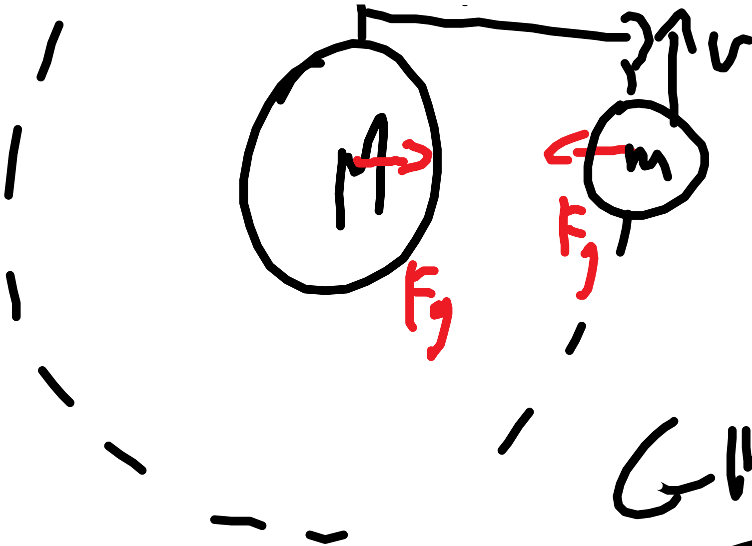
We are looking at Newtonian gravity for now assuming circular orbits.

$F_g = GMm/r^2$

gravitational pull, F_g, in newtons is proportional to the masses, m and M, and inversely proportional to the distance between the centre of the masses, r.

G - universal gravitational constant $6.67 \times 10^{-11} Nm^2/kg^2$ everywhere in the universe





$$\boxed{F_g = F_c}$$

$$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{r} = \frac{\cancel{m}4\pi^2 r}{T^2}$$

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$GMT^2 = 4\pi^2 r^3$$

$$\boxed{\frac{T^2}{r^3} = \frac{4\pi^2}{GM}}$$ KTL

Central mass
kepler - Sun

Kepler - Sun

Gravity

History - Aristotle - natural resting points. Earth and water go down, air and fire go up. Things with more earth (mass) fall faster.

Ptolemy - planets revolved around Earth in epicycles. (circle in circle)

Copernicus - Sun centre of solar system.

Galileo - things fall at the same rate regardless of mass if air resistance is negligible. We don't feel the motion of the earth because of our inertia.

Kepler - Laws of motion of Celestial Bodies.

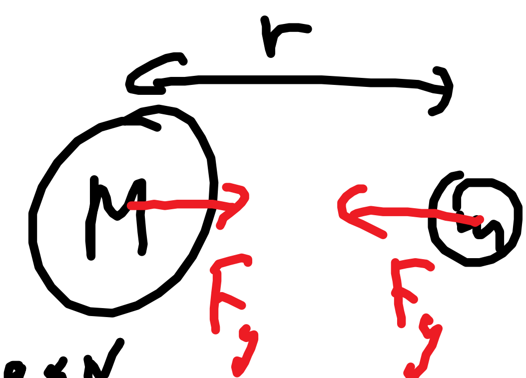
- Planets move in elliptical orbits with the sun at focii.
- The line between the planet and the sun sweeps out equal areas in equal times.
- ratio of the period of revolution of the

planets squared to their average radii cubed is a constant T^2/r^3 = k constant

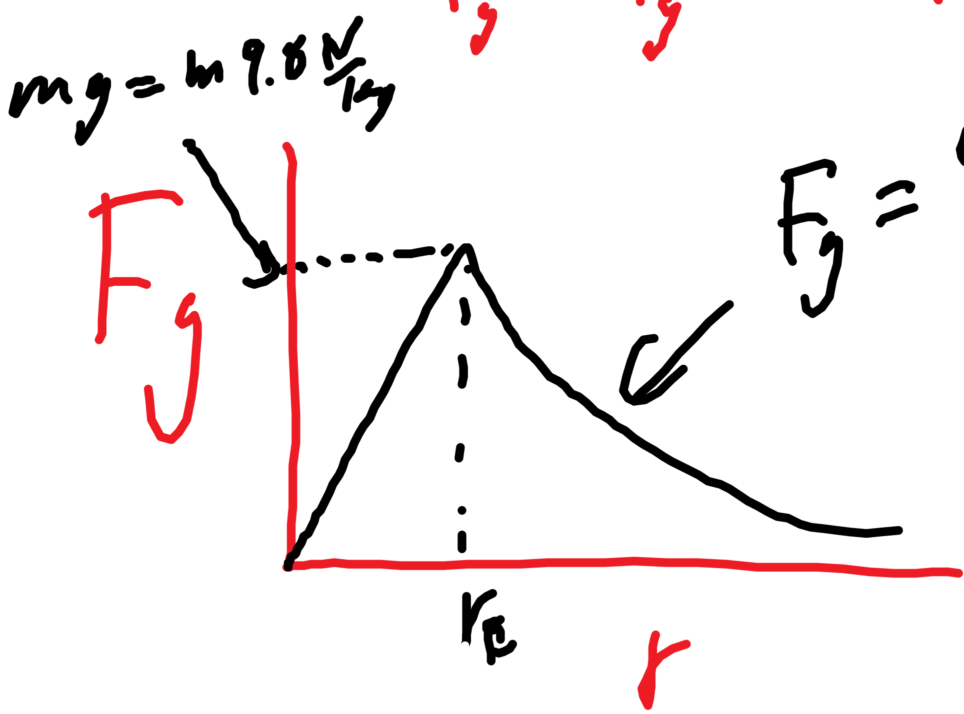
Newton - Gravity is a force that attracts all masses to each other. Everything in the universe is attractive, more mass=more attraction, more distance between the mass= less attraction

$F_g \propto mM$

$F_g \propto \frac{1}{r^2}$



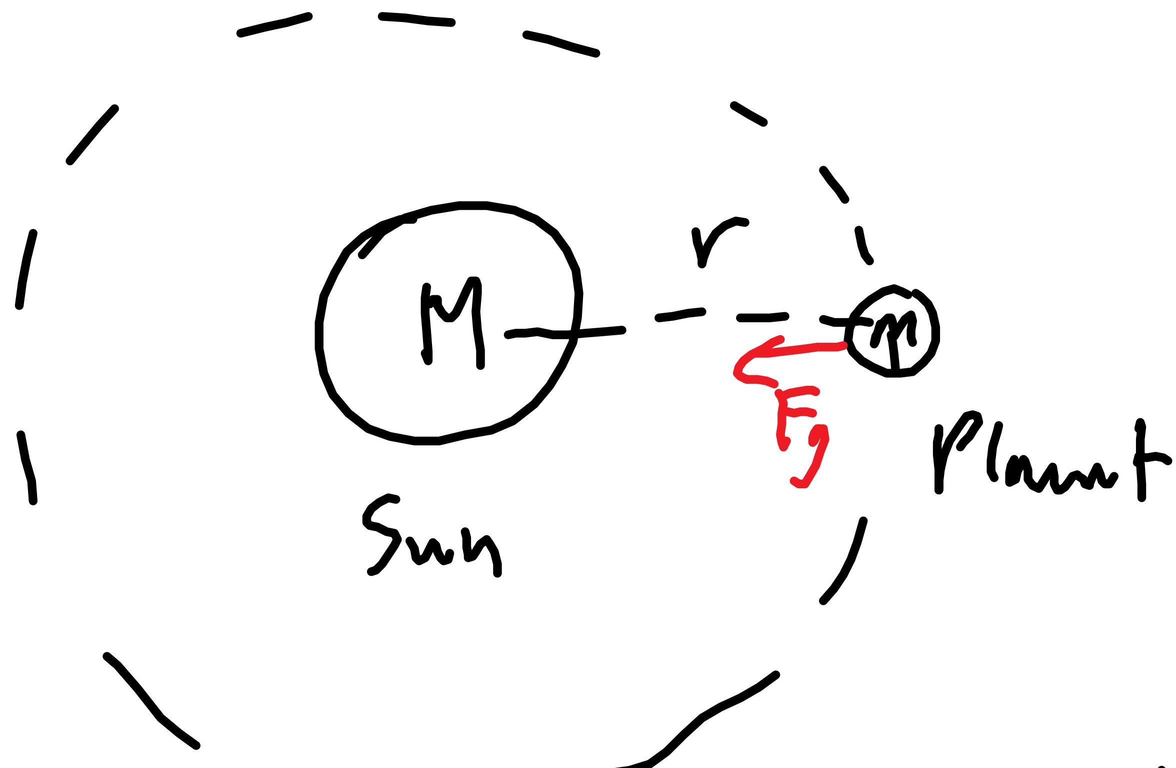
$F_g = \frac{GMm}{r^2}$



F_g=GMm/r^2

M and m are any two masses, in kg.
r is the distance between the centres of the two masses, in m.
G is universal gravitational constant $6.67 \times 10^{-11} Nm^2/kg^2$

Orbits:



$$\boxed{F_g = F_c}$$ * orbits

$$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{r} = \frac{\cancel{m}4\pi^2 r}{T^2}$$

$$g = \frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$$

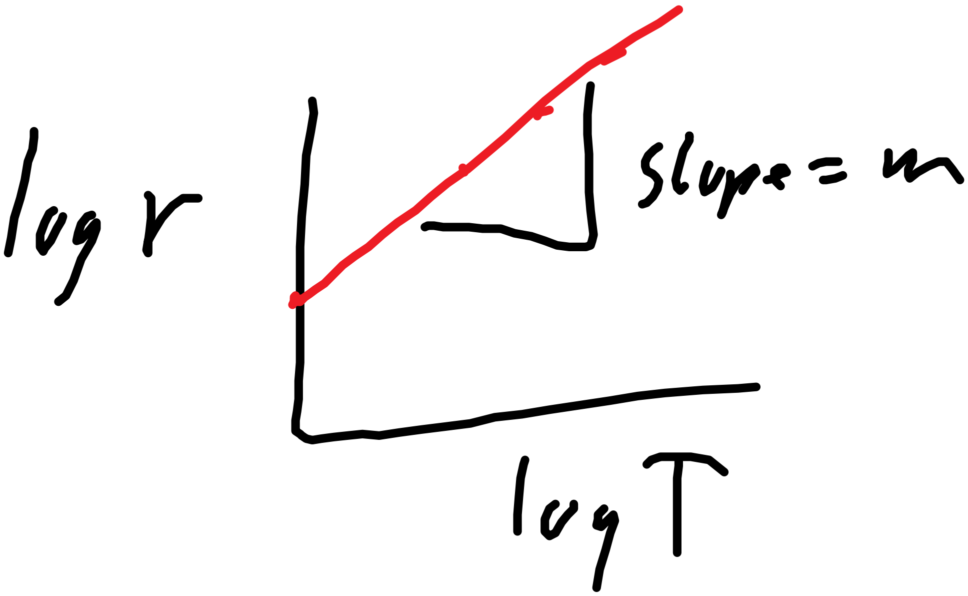
$$GMT^2 = 4\pi^2 r^3$$

$$\boxed{\frac{T^2}{r^3} = \frac{4\pi^2}{GM}}$$

mass of the sun if we are talking about planets

Einstein - gravity is curvature of spacetime.

Lab



$$\log r = m \log T + b$$

relate to orbits: $F_c = F_g$

$$\frac{\cancel{m}\, 4\pi^2 r}{T^2} = \frac{G M \cancel{m}}{r^2}$$

$$r^3 = \frac{GM}{4\pi^2} T^2$$

$$3\log r = \log\frac{GM}{4\pi^2} + 2\log T$$

$$\log r = \frac{1}{3}\log\frac{GM}{4\pi^2} + \frac{2}{3}\log T$$

1.98×10^{30} kg

Energy

is conserved

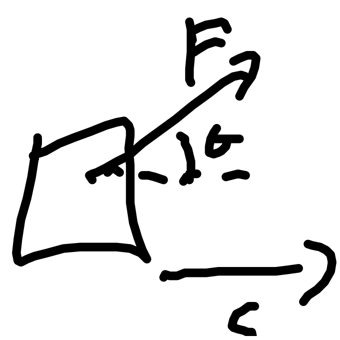
energy of motion, Kinetic Energy

$$E_K = \frac{1}{2}mv^2$$

derived from Work-Energy Theorem

$$W = \Delta \text{Energy}$$

$$W = Fs\cos\theta$$



↑
only for Force is constant

$W = \int F\, ds$ = area under F-s graph

elastic force, $F_{elastic}$

$-F_{elastic}$

area = $\frac{1}{2} F x$

$F = -kx$

Elastic energy $= \frac{1}{2} k x^2$

k is elastic constant

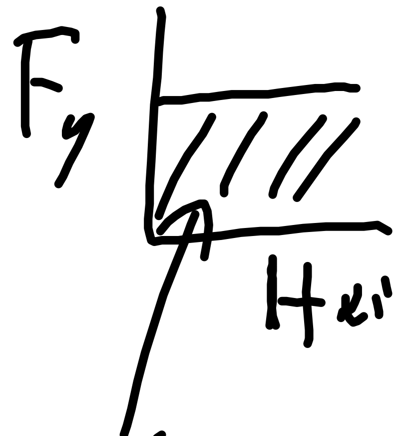
Thermal energy → Heat = ~~[illegible]~~

= Δ internal energy = $\Delta U = Q$

$Q = mc\Delta T = mH$

$$Q = \underbrace{mc\Delta T}_{\text{warming}} = \underbrace{mH}_{\text{change state}}$$

Gravitational Energy



Near Earth

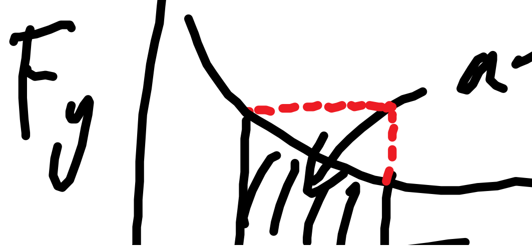
$F_g = mg$

$g = 9.81\ N/kg$

area $= \Delta E_g = F_g H$

$$\boxed{E_g = mgh}$$

Far from Earth



area $= \int F_g\, dr$

$$\int_{}^{r} GMm\, r^{-2}\, dr = \frac{-GMm}{r_f} - \frac{-GMm}{r_i}$$

$$\boxed{E_g = -\frac{GMm}{r}}$$

1. You pull a 45.0 kg bale of hay with 140 N of force at 28.0º above the horizontal. If the coefficient of friction is 0.30, determine

a) horizontal component of the applied force

b) normal force (not = mg)

c) the acceleration of the bale

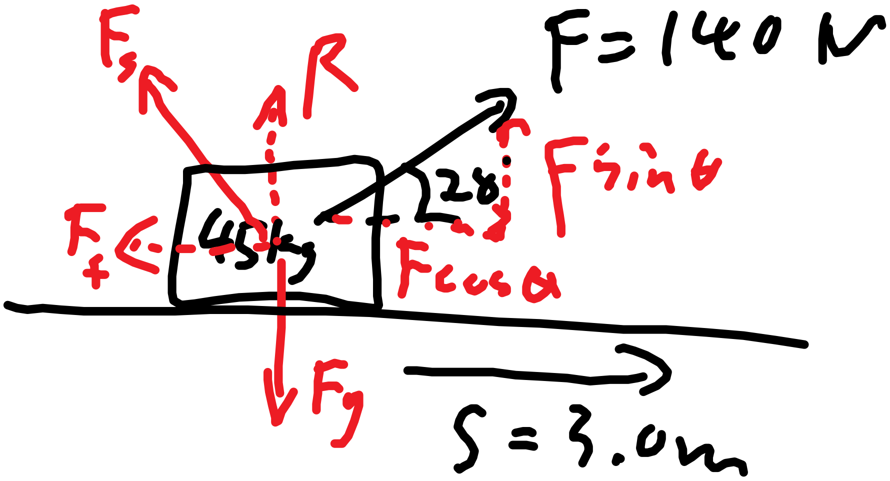
d) the work done by you after you pull it 3.0m

e) the work done by friction

f) the kinetic energy of the bale

g) You drop the bale of hay onto a spring

from a height of 2.0m to the top of the spring. If the spring constant is 3200 N/m, what is the maximum compression of the spring?



a) $F\cos\theta = 140\cos 28$

$= 124\text{ N}$

b) $R + F\sin\theta = F_g$

$R = 45(9.81) - 140\sin 28$

$R = 376\text{ N}$

c) $F_{net} = ma = \Sigma F$

$a = 124\text{N} - 0.3(376\text{N})$

$$a = \frac{124N - 0.3(376N)}{45kg}$$

$a = \cancel{0.249}$ $\boxed{0.249\ m/s^2}$

d) $W = F_s \cos\theta = \quad J = N \cdot m$

$140N(3m)\cos 28^\circ$

$= 372 = \boxed{3.7 \times 10^2\ J}$

e) $W = F_f \times s = -112.8 \times 3$

$= \boxed{-3.4 \times 10^2\ J}$

↑ if F is opposite s

$\Delta E_k = mas$

$= F_{net}\, s$

f) 45x0.247x3=33J

g) $E_g = E_{elastic}$

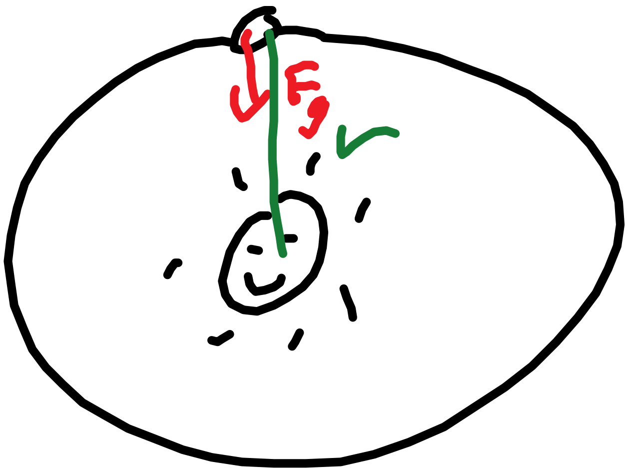
$mg(h+x) = 1/2kx^2$

$45 \times 9.8(2.0 + x) = 1/2\ 3200\ x^2$

> 2

45x9.8(2.0 +x) =1/2 3200 x^2
x= quadratic solution

Lab



orbits $\boxed{F_c = F_g}$

$$\frac{\cancel{m} 4\pi^2 r}{T^2} = \frac{GM\cancel{m}}{r^2}$$

$$r^3 = \frac{GM}{4\pi^2} T^2$$

$$\log(r^3) = \log\left(\frac{GM}{4\pi^2} T^2\right)$$

$$3\log r = \log \frac{GM}{4\pi^2} + 2\log T$$

$$\log r = \underbrace{\frac{1}{3}\log\frac{GM}{4\pi^2}}_{\text{y-int.}} + \underbrace{\frac{2}{3}}_{\text{slope}}\log T$$

$$M_s = 1.98\times10^{30}\,\text{kg}$$

$$\frac{1}{3}\log\left(\frac{6.67\times10^{-11}\,(1.98\times10^{30})}{4\pi^2}\right)$$

$$= \boxed{6.17}$$

Energy

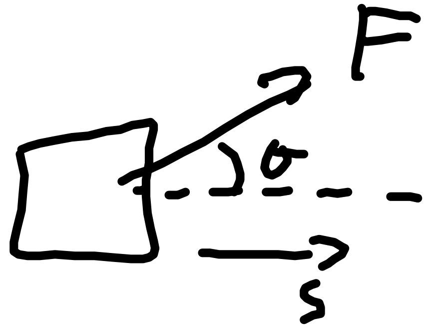
Work Energy Theorem

W=change in energy

$W = Fs\cos\theta$

F is force

s is displacement



θ is the angle between F and s.

if F is not constant, W = area under F-s graph = $\int F ds$

units: Joule, J = Nm

Energy is conserved. It only changes form (mass is a form of energy $E=mc^2$)

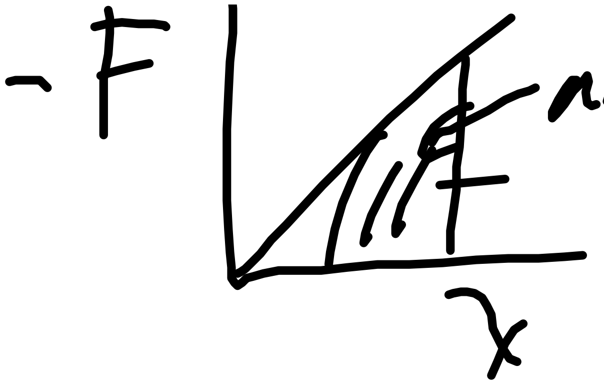
forms of energy

Energy of motion, kinetic energy E_k

$E_k = 1/2\ mv^2$ derive from $w = F_{net}s$

elastic energy = Fs   but F = -kx

= F | [illegible] = W = Δ Energy



area = W = Δ energy

$$W = \frac{1}{2} x(-F)$$

$$W = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2$$

$$\boxed{\text{Energy} = \frac{1}{2} k x^2}$$

IB thermal Energy = Heat = Q

= change in internal energy = ΔU

$$Q = \underbrace{mc\Delta T}_{\text{Heating}} = \underbrace{mH}_{\text{change of state}}$$

Gravitational Energy, E_g

area = $W = \Delta E_g$

Near Earth

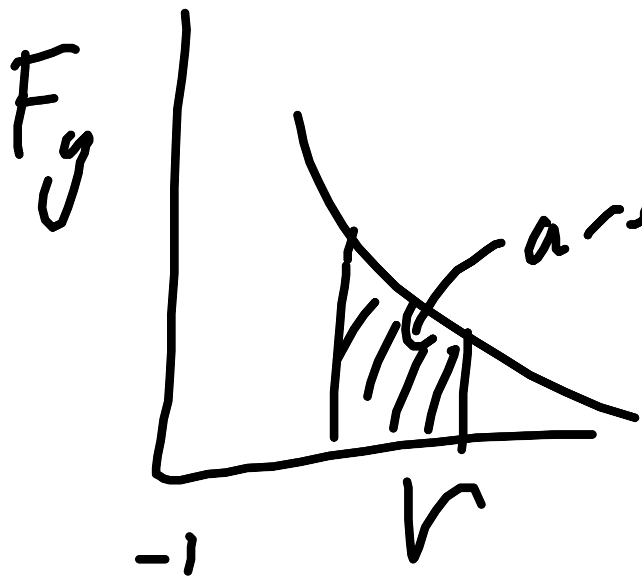
Near Earth

$F_g = mg$

$g = 9.81\ N/kg$

$\Delta E_g = F_g \Delta h$

$$\boxed{E_g = mgh}$$



area $= W = \Delta E_g = \int F_g\, dr$

$$\int_{r_i}^{r_f} GMm\, r^{-2}\, dr$$

$$\frac{dx^{2}}{dx} = 2x \qquad \int x\, dx = \frac{1}{2} x^2$$

-1 +1

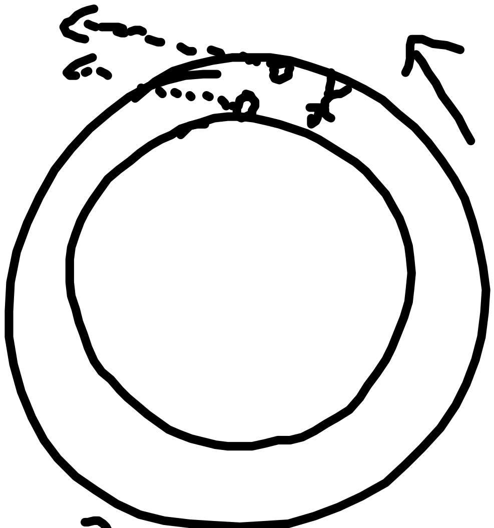
$$\boxed{-\frac{GMm}{r_f} - -\frac{GMm}{r_i} = \Delta E_g}$$

$$\boxed{E_g = -\frac{GMm}{r}}$$

relative to $E_g = 0$ at $r = \infty$

Friday, November 20, 2015
8:44 AM

Review-
p96 - General Problems 55, 56, 59, 63
p123 52, 54, 56, 57, 61,62
p147 62, 64, 67
Q56 p123

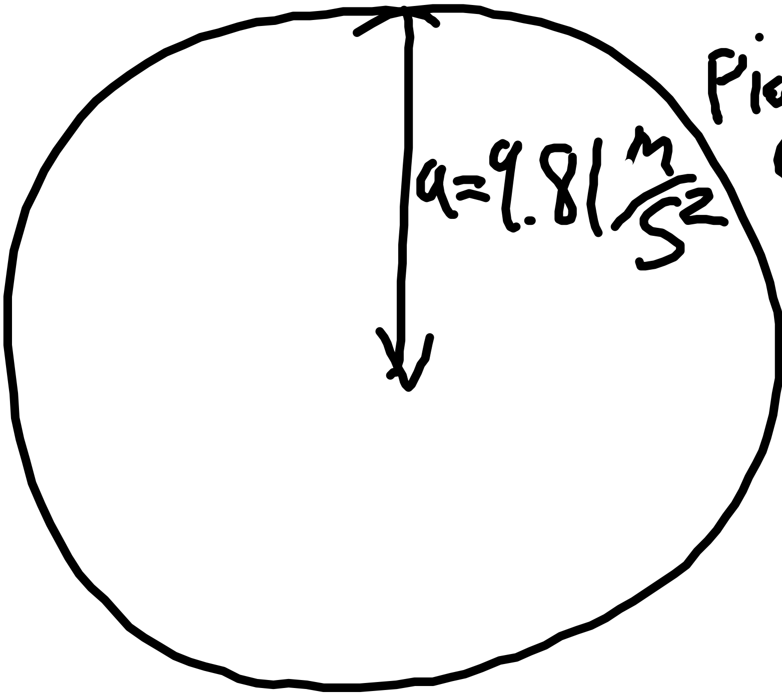


$a_c = g \quad \frac{v^2}{r} = 9.81 \quad v = 77\ m/s$

$\frac{1200\pi\ m}{77\ m/s} = 49\ s$

$24 \cdot 60 \cdot 60 / 49 = 1800\ R/d$

P123 Question 61



Planet earth AKA Mongo

Given radius of earth

$r = 6.38 \times 10^6\ m$

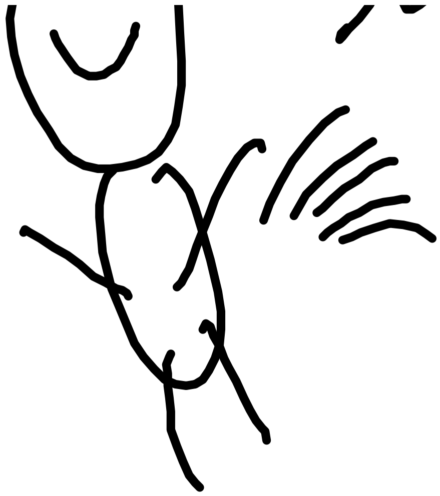
$a = \frac{\vec{v}^2}{r} \Rightarrow 9.81 = \frac{\vec{v}^2}{r}$

$\therefore \vec{v} = 7911\ m/s$

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$\bigcirc = \frac{2\pi r}{\text{Time}}$

Solve for time



$t = 1.41$ hours

P123 52

$F_{c\,max} = 1200N$

$1200N = \frac{mv^2}{r}$

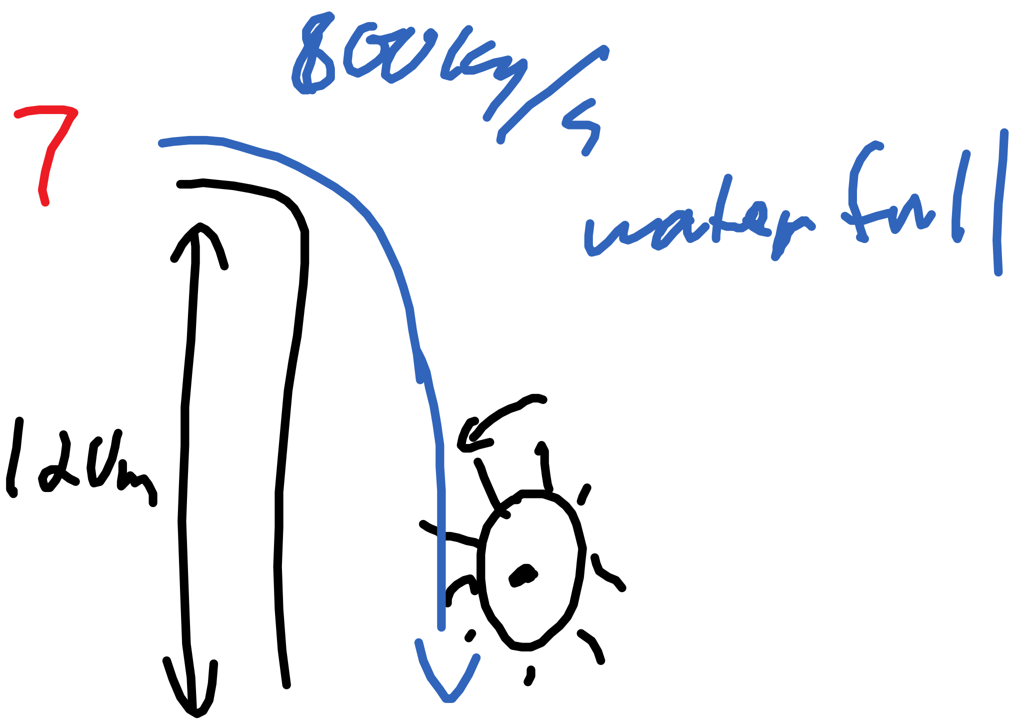
$v = 8.23$ m/s

$1200N - mg = F_c$

$1200 - (85)(9.81) = \frac{85v^2}{4.8m}$

$\boxed{v = 4.5\ m/s}$ $\boxed{4.5}$

Q 47



$\frac{1}{2} m v^2 = m g h$

$v = \sqrt{2 g h} = 48.5\ m/s$

b) $P = ?$

80% of v lost

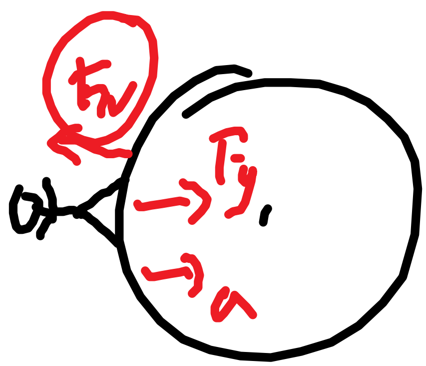
12% lost as thermal

$$\left[\frac{1}{2} m \left\{(0.8)(48.3)\right\}^2\right] \times 0.88$$

?

Q54.

a = ?



apparent weight

$F_g - F_N = F_c$

$$mg - F_N = m\frac{4\pi^2 r}{T^2}$$

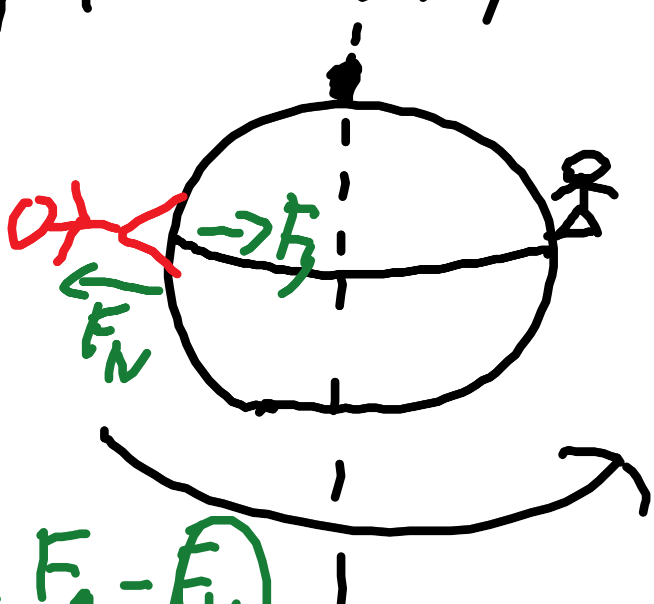
$$F_N = m\left[\left(\frac{4\pi^2(6.38\times10^6\,m)}{(24\times3600\,s)^2}\right) - g\right]$$

$0.0\,3374\ m/s^2$

$9.82 - 0.06 = \underline{9.76}$

Review-
p96 - General Problems 55, 56, 59, 63
p123 52, 54, 56, 57, 61,62
p147 62, 64,
6.38 x 10^6m

p123 Q54



g

$a_c = \frac{v^2}{R}$

$F_c = F_g - F_N$

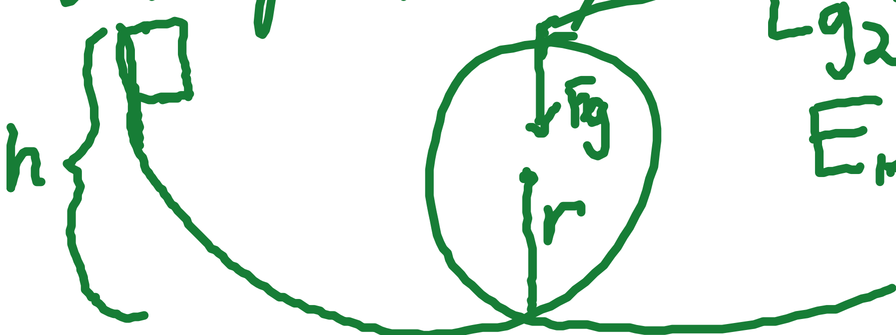
$a_c = \frac{v^2}{R} = \frac{4\pi^2 R}{(1\,day)^2}$

$$V = \frac{2\pi R}{1\,\text{day}} \qquad (1\,\text{day})^2$$

$$= 0.0337\ \text{m/s}^2$$

$$\frac{0.0337\ \text{m/s}^2}{g} \times 100\% = 0.344\%$$

6) p 147



$E_{g_2} = mg(2r)$

E_K

$F_g = F_c$

$\cancel{m}g = \frac{v^2}{r}\cancel{m}$

$E_{g_1} = mgh$

$E_K = 0$

$E_{g_1} = E_{g_2} + E_K$

$\cancel{m}\cancel{g}h = \cancel{m}\cancel{g}2r + \frac{1}{2}mv^2$

$+ \frac{1}{2}\cancel{g}r$

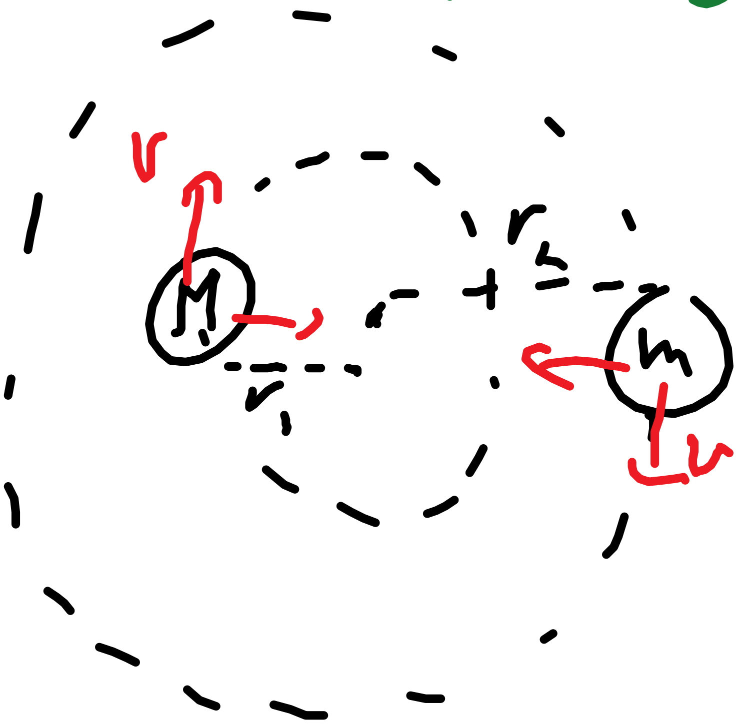
$h = 2r + \frac{1}{2}r$

$= 2.5r$

work in = work out

$$F_{feet} \times d_{feet} = Force_{tire} \times d_{tire}$$

$$F_p \times d_p = mg\sin\theta\left(d_p \, \frac{T2}{l9} \, \frac{\cancel{2\pi} r_f}{\cancel{2\pi} r_p}\right)$$



$$\frac{GMm}{(r_1+r_2)^2} = \frac{\cancel{M}4\pi^2 r_1}{T^2}$$

$$= \frac{m4\pi^2 r_2}{T^2}$$

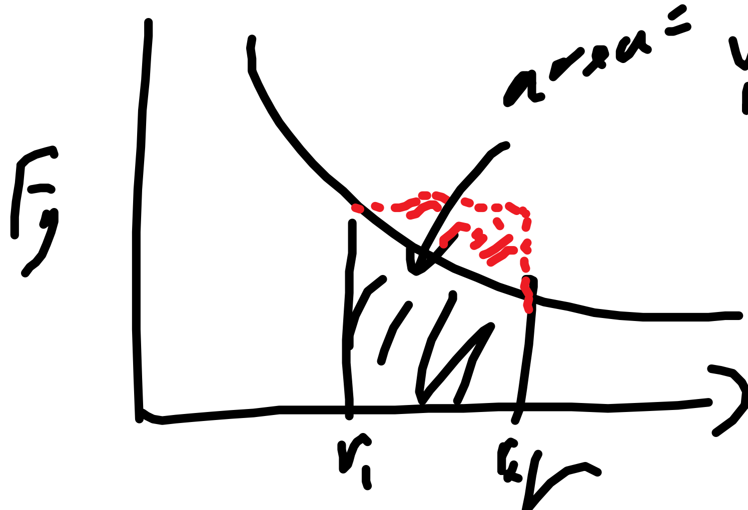
Thursday, November 26, 2015
8:47 AM

Gravitational Energy

Near Earth

E_g=mgh   g=9.81 N/kg

$$\text{area} = \int_{r_1}^{r_a} F_g \, dr$$



Universal

E_g=-GMm/r

relative to

Eg=0

at r=infinity

$$= -GMm\left(\frac{1}{r_a} - \frac{1}{r_1}\right)$$

Set reference point $r \to \infty$

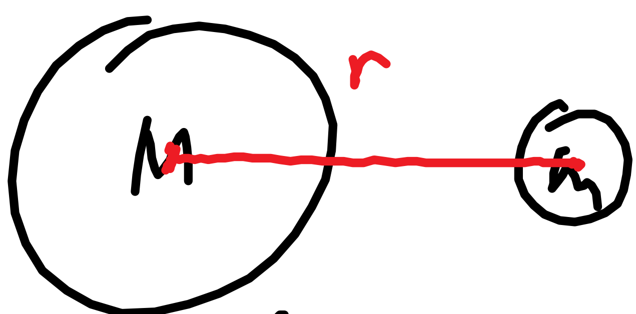
$$E_g \to 0$$

Definition of gravitational energy - work required to move a mass, m, from infinity to r.

Energy due to the gravitational field relative to 0 at r=infinity.

eg. A 2.0×10^3 kg space capsule leaves Earth. How much energy in fuel is required (ignore air resistance) to

a) go up 50.0 m above Earth's surface
b) go up 400.0 km above Earth
c) go into stable orbit 400.0 km above the Earth's surface
d) escape the Earth's gravitational field
e) graph Energy vs r (include total energy, gravitational energy and kinetic energy) of the space ship for
  - projectile with i) escape energy ii) half escape energy iii) double escape energy
  - satellites in stable orbits (general shape)



$r = 6.38 \times 10^{6}\,m$

$M = 5.98 \times 10^{24}\,kg$

a) $E_g = mgh = 2000\,kg(9.81)(50)$

$= 9.8 \times 10^{5}\,J$

b) $\Delta E_g = -GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

$-6.67 \times 10^{-11}(5.98 \times 10^{24})(2000)\left(\frac{1}{6.78 \times 10^{6}} - \frac{1}{6.38 \times 10^{6}}\right)$

Gravitational Energy, E_g

related to your position in a gravitational field

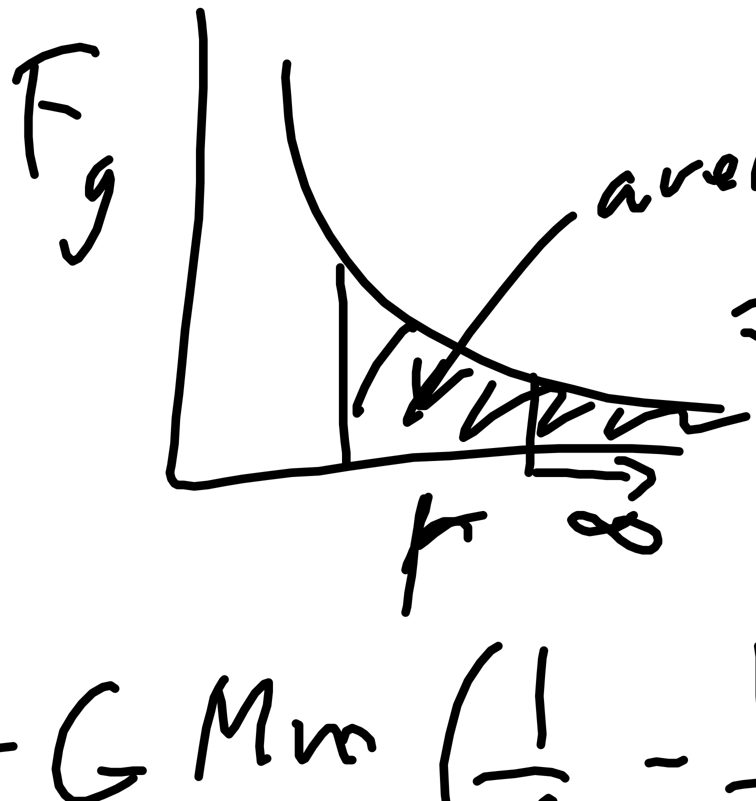
uniform field, g is constant over the change in height

eg. near Earth, g=9.81 N/kg

E_g= mgh

non-uniform field -

work <u>against</u> gravity



$$\text{area} = \Delta E_g = W_g$$

$$= \int_{r_1}^{r_2} F_g \, dr$$

$$-GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \Delta E_g$$

$$-GMm\left(\frac{1}{r_2}-\frac{1}{r_1}\right)=\Delta E_g$$

$$\boxed{E_g=-\frac{GMm}{r}}$$

relative to $E_g \to 0$

$r \to \infty$

infinity

Define gravitational energy, E_g as the work required to move a mass m from infinity to r.

orbits $F_c = F_g$

$$\frac{\cancel{m}v^2}{} = \frac{GM\cancel{m}}{}$$

$$\cancel{x} - \frac{\quad}{r^2}$$

$$v = \sqrt{\frac{GM}{r}}$$ orbital speed

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}$$

$$E_K = \frac{1}{2}|E_g|$$ orbits

Earth. How much energy in fuel is required (ignore air resistance) to

a) go up 50.0 m above Earth's surface

$E_g = mgh = \boxed{981\ kJ}$

b) go up 400.0 km above Earth

$\Delta E_g = -GMm(1/r_f - 1/r_i)$

$-6.67 \times 10^{-11}(5.98 \times 10^{24})(2000)$

$\times (1/6.78 \times 10^6 - 1/6.38 \times 10^6)$

$-(\underline{1.1766 \times 10^{11} J} - \underline{1.25036 \times 10^{11} J})$

$= 7.38 \times 10^9 J$

c) go into stable orbit 400.0 km above the Earth's surface

answer from b and add E_k

$F_c = F_g$

$mv^2/r = GMm/r^2$

$v^2 = GM/r$

$E_k = 1/2\ mv^2 = 1/2\ GMm/r = 1/2\ |E_g|$

for a stable orbit

$E_k = 1/2\ (6.67 \times 10^{-11})(5.98 \times 10^{24})$

$(2000) / 6.78 \times 10^6)$

$= 1.1769 \times 10^{11} J/2 = 5.88 \times 10^{10} J$

$7.4 \times 10^{9} J + 5.88 \times 10^{10} J = 6.62 \times 10^{10} J$

ΔE_g $\quad\quad$ E_k $\quad\quad$ E_t

d) escape the Earth's gravitational field

E_k > or = ΔE_g

Egf = 0 at r = infinity

E_k = Egf - Egi

Ek = 0 - (-GMm/r_i)

Ek = GMm/r =

$(6.67 \times 10^{-11})(5.98 \times 10^{24})(2000) / 6.38 \times 10^{6})$

$= 1.25 \times 10^{11} J$

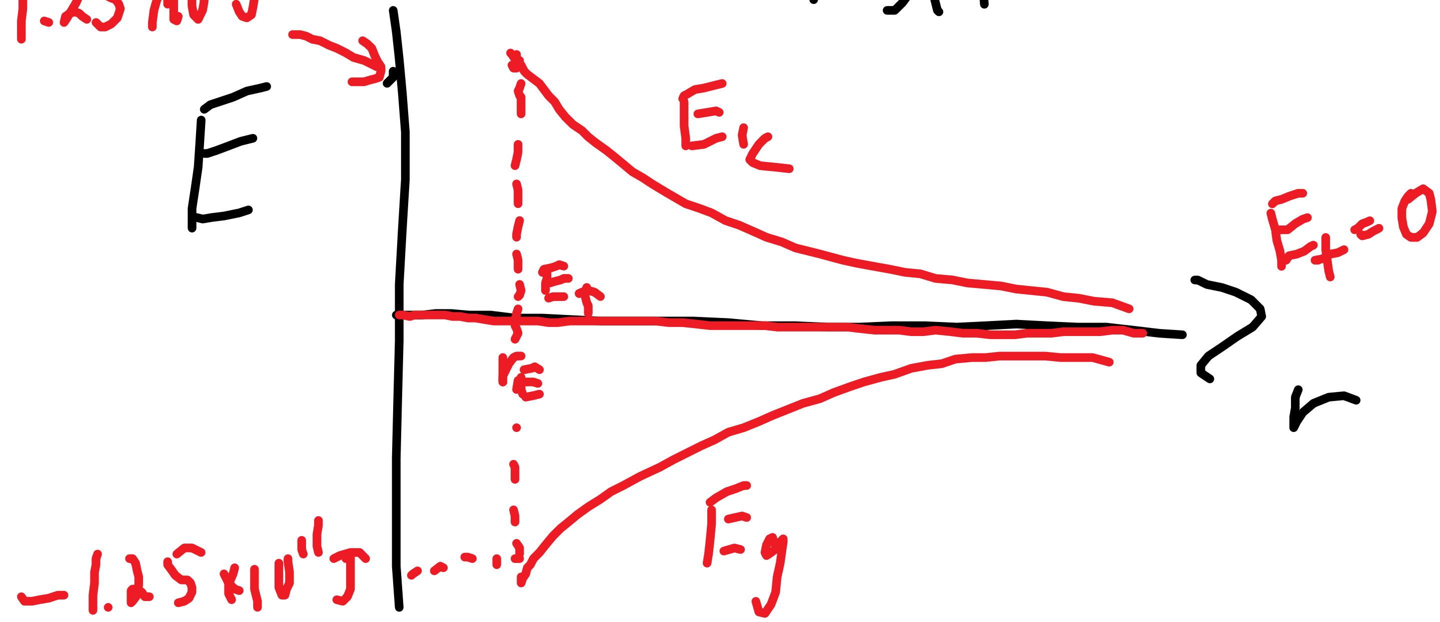
escape speed 1.12×10^{4} m/s

e) graph Energy vs r (include total energy, gravitational energy and kinetic energy)
of the space ship for

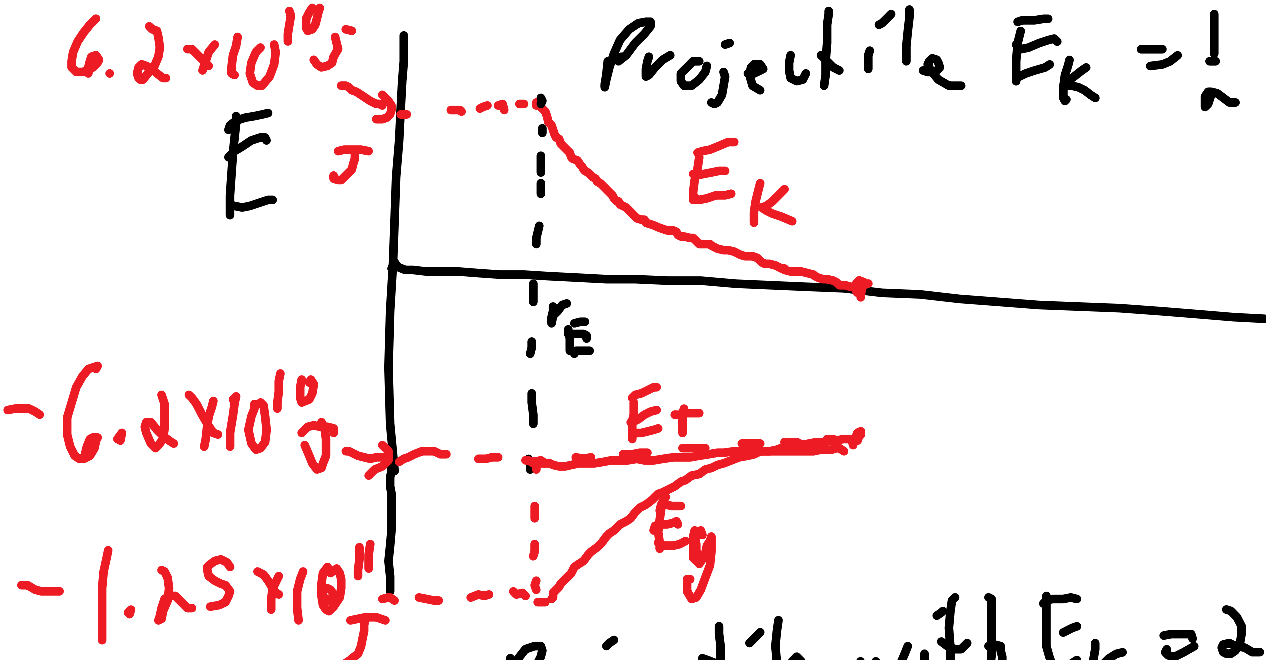
- projectile with i) escape energy ii) half escape energy iii) double escape energy
- satellites in stable orbits (general

shape)

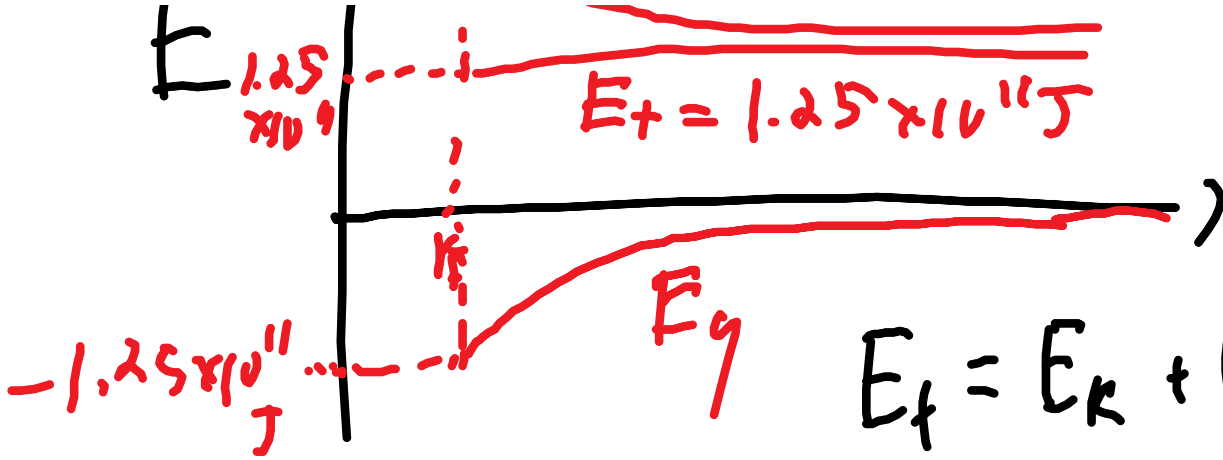
$E_{K_i} = E_{escape} = |E_{g_i}|$



Projectile $E_K = \frac{1}{2} E_{escape}$



Projectile with $E_K = 2 E_{escape}$



$E_t = E_K + E_g$

what about stable orbits

$F_c = F_g$

$\frac{mv^2}{\cancel{r}} = \frac{GMm}{r^{\cancel{2}}}$

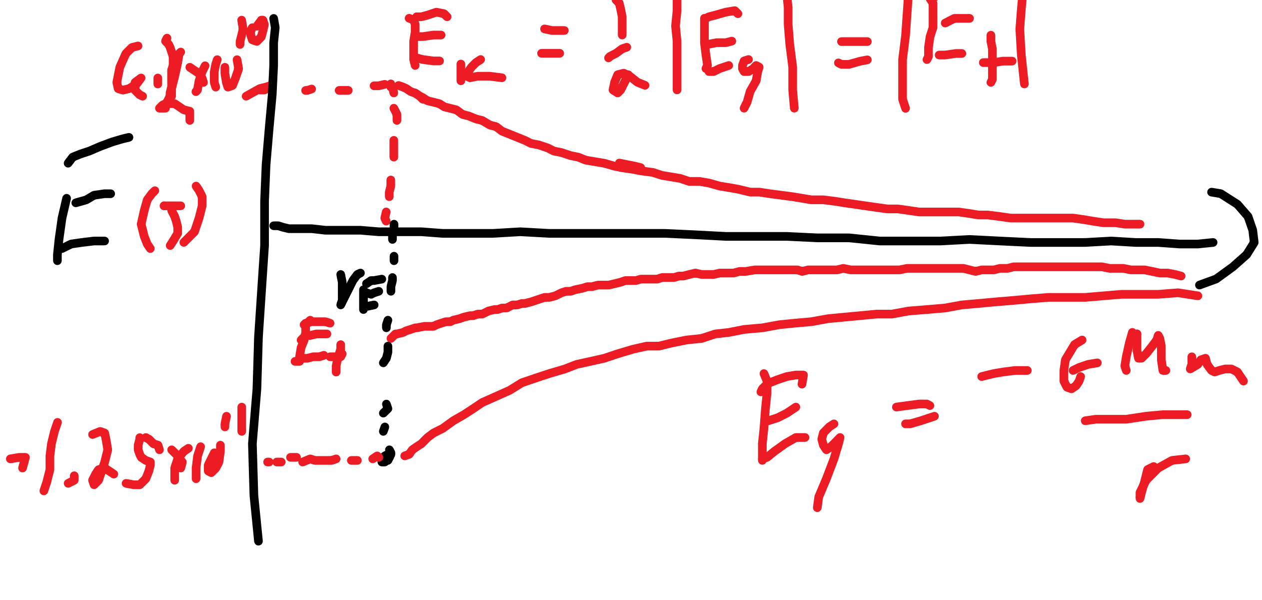
$E_K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r} = \frac{1}{2}|E_g|$

$E_t = E_K + E_g$

$E_t = \frac{1}{2}|E_g| + E_g$

$E_t = \frac{1}{2}E_g$

$E_K = \frac{1}{2}|E_g| = |E_t|$



$$E_t = E_k + E_g$$

$$E_t = \frac{1}{2}|E_g| + E_g$$

$$\boxed{E_t = \frac{1}{2}E_g}$$ for orbits